Your name: ______________________

8.20 Special Relativity

R. L. Jaffe

IAP
January 27, 1993

FINAL EXAM

7:00 – 8:30 p.m., Tuesday, January 26

NOTES:
Grades will be sent in on Friday afternoon at 4:30 pm.

- You have from 10:00 – 12:00 pm.
- Closed book, no notes.
- Do all your work on the exam.
- There are five problems.
  Point values are shown.
- Try them all.
- Use of calculators is allowed.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	

INFORMATION

- Lorentz Transformations:

Let $W_\mu = (W_0, W_1, W_2, W_3)$ be any four-vector)

$$\begin{cases} W_0' = \gamma(W_0 - \beta W_1) \\ W_1' = \gamma(W_1 - \beta W_0) \\ W_2' = W_2 \\ W_3' = W_3 \end{cases} \qquad \begin{array}{l} W_0 = \gamma(W_0' + \beta W_1') \\ W_1 = \gamma(W_1' + \beta W_0') \\ W_2 = W_2' \\ W_3 = W_3' \end{array}$$

(when S' moves at speed v along the $\hat{x}$ direction in S).

$$\beta = \frac{v}{c} \ , \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

- Relativistic energy and momentum

$$E = \sqrt{\vec{p}^2 c^2 + m^2 c^4} \ , \qquad \vec{p} = m\vec{v}\gamma \ , \qquad E = mc^2\gamma$$

- Velocity addition:

$$u_x' = \frac{u_x - v}{1 - u_x v/c^2} \qquad\qquad u_x = \frac{u_x' + v}{1 + u_x' v/c^2}$$

$$u_y' = \frac{u_y}{\gamma(1 - u_x v/c^2)} \qquad\qquad u_y = \frac{u_y'}{\gamma(1 + u_x' v/c^2)}$$

(when S' moves with speed v along the $\hat{x}$ direction in S).

- Light: $E = h\nu$, $\lambda\nu = c$.
- Doppler shift:

  Longitudinal: $\nu = \nu_0\sqrt{\frac{1-\beta}{1+\beta}}$ when source recedes at $v = \beta c$ from observer.

  Transverse: $\nu = \sqrt{1-\beta^2}\,\nu_0$ when source moves at $v = \beta c$ transverse to line of sight from observer.

- Units:

$$1\,\text{electron Volt} = 1.6 \times 10^{-19}\text{joule} = 1.6 \times 10^{-12}\,\text{erg}$$

$$10^6\,\text{eV} = 1\,\text{MeV}$$

$$1000\,\text{MeV} = 1\,\text{GeV}$$

$$1\,\text{joule} = 0.239\,\text{cal}$$

$$1\,\text{Btu} = 1060\,\text{joule}$$

$$1\,\text{joule} = 1\,\text{Newton-meter}$$

$$1\,\text{watt} = 1\,\text{joule/sec}$$

$$1\,\text{Newton} = 1\,\text{kg meter/sec}^2$$

- Quadratic Formula:

$$ax^2 + bx + c = 0 \longrightarrow x = \frac{1}{2a}\left(-b \pm \sqrt{b^2 - 4ac}\right)$$

- Binomial expansion:

$$(1+a)^b = 1 + ba + \frac{b(b-1)}{2}a^2 + \frac{b(b-1)(b-2)}{6}a^3 + \ldots$$

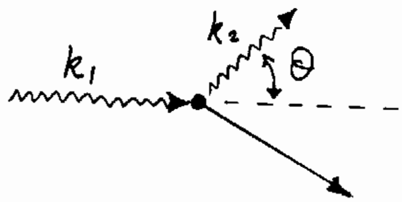
- Some constants of Nature:

$$c = 300\,\text{km/millisec}$$
$$h = 6.626 \times 10^{-34}\,\text{joule/sec}$$
$$G = 6.672 \times 10^{-11}\,\text{Newton} - m^2/\text{kg}^{-2} \quad \text{(Newton's constant)}$$
$$L_\odot = 3.83 \times 10^{26}\,\text{joules/sec} \quad \text{(Solar luminosity)}$$
$$M_\odot = 1.989 \times 10^{30}\,\text{kg} \quad \text{(Solar Mass)}$$

Proton mass :	$M_p c^2 = 939\,\text{MeV}$
Pion mass :	$M_\pi c^2 = 140\,\text{MeV}$
Muon mass :	$M_\mu c^2 = 106\,\text{MeV}$
Electron mass :	$M_e c^2 = 511\,\text{KeV}$
Kaon mass :	$M_K c^2 = 494\,\text{MeV}$
Lambda mass :	$M_\Lambda c^2 = 1116\,\text{MeV}$

Problem 2: (20 points)

Compton scattering is the scattering of light by electrons. Derive the Compton formula relating the shift of wavelength of light to the scattering angle.



$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Problem 3: (20 points)

A particle physicist would very much like to study the "top-quark" which is believed to have a mass, m_t, at least 150 times the mass of the proton, M_p. The simplest reaction she can think of is

$$\overline{P} + P \rightarrow \bar{t} + t \quad .$$

P is a proton, $\overline{P}$ is an antiproton with the same mass as the proton. t is a top quark, $\bar{t}$ is an anti-top quark with the same mass as the top quark.

a) (8 points) Physicist 1 (she) proposes to build two accelerators, one for protons, the other for antiprotons and collide the two beams head-on in the laboratory. If accelerators cost $\$1 \times 10^6$ per GeV of energy, how much would her equipment cost?

b) (12 points) A government agency decides two accelerators are too expensive and solicits proposals to study the same reaction $(\overline{P} + P \rightarrow \bar{t} + t)$ using only one accelerator. Physicist 2 (he) submits a proposal to accelerate anti-protons and direct them onto a proton target at rest in the lab. At $\$1 \times 10^6$ per GeV, how much would his equipment cost? What would you recommend to the government agency?

Problem 4: (20 points)

Normal nuclei consist of protons and neutrons. The lambda (Λ) particle resembles protons and neutrons in many ways and physicists like to study nuclei with a Λ-particle added in. Lambda particles can be made in the reaction

$$K + p \rightarrow \pi + \Lambda \qquad (*)$$

Here K is a kaon, π is a pion, p is a proton. Their masses are given in the information sheet.

Many years ago, MIT's Herman Feshbach pointed out that if a kaon of just the proper energy (E^*) struck a proton at rest in a nucleus, then the Λ *would be created at rest* and this would be a very effective way to put a Λ into the nucleus. The point of this problem is to find E^*.

a) (3 points) Write conservation of energy and momentum for reaction (*) as a four-vector equation.

b) (7 points) Assuming both the initial proton and final Λ to be at rest, find an algebraic formula for the energy of the kaon, E^*. [Hint: Use invariants.]

c) (4 points) Substitute numbers into your result to find E^* in units of MeV. What is the *kinetic* energy of the kaon?

d) (6 points) If the mass of the pion were too large, this process would not work. What is the maximum pion mass allowed?

Problem 5: (20 points)

Neutrinos are thought to be massless particles. When the supernova was observed in the Large Magellanic Cloud (LMC) in 1987, a burst of neutrinos was detected within one hour of the first observation of the supernova with visible light. The neutrinos had energies of about 5 MeV. The distance from Earth to the LMC is about 50,000 light years.

a) (10 points) Assuming the supernova explosion emitted visible light and neutrinos simultaneously, what upper limit does this data place on the possible mass (m_ν)of the neutrino? [You can express your answer in terms of the neutrino rest energy, $m_\nu c^2$.]

b) (10 points) If the neutrinos are massive and are unstable they could decay in transit and would not have made it to the Earth. Comparison between observation and the theory of supernovas suggest that at least half the number of neutrinos expected from the theory were detected. What limit does this place on the neutrinos "half-life" — the time (in the neutrino's rest frame) required for half to decay? First express your answer in terms of the neutrino rest energy, $m_\nu c^2$ and the information given in the problem. Then suppose $m_\nu c^2$ was equal to the upper limit you found in part a) and give a numerical result.