MASSACHUSETTS INSTITUTE OF TECHNOLOGY

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Quiz 1

PART I (42 points)

For each one of the statements below, state whether it is true or false.

You may write the answers on this sheet and hand it in; explanations or comments are not needed and will not be taken into account.

In questions 2-14, assume that we are dealing with a linear programming problem in standard form, under the usual assumption that the rows of $\mathbf{A}$ are linearly independent.

Correct answers: 3 points
Wrong answers: -1 **points**
No answer: 0 points

1. Let $P = \{\sum_{k=1}^{K} \lambda_k \mathbf{x}^{\mathbf{k}} \mid \lambda_{\mathbf{k}} \geq \mathbf{0},\ \sum_{\mathbf{k=1}}^{\mathbf{K}} \lambda_{\mathbf{k}} = \mathbf{1}\}$, where $\mathbf{x}^{\mathbf{1}}, \ldots, \mathbf{x}^{\mathbf{K}}$ are given vectors. Then the set of extreme points of P is $\{\mathbf{x}^{\mathbf{1}}, \ldots, \mathbf{x}^{\mathbf{K}}\}$.

2. In an iteration of the simplex method, if the variable exiting the basis is zero prior to the iteration, then the new basic solution is degenerate.

3. While running the simplex method, once the reduced cost of some variable becomes nonnegative, it remains nonnegative throughout the algorithm.

4. The optimal cost of the auxiliary problem solved in Phase I of the simplex method cannot be $-\infty$.

5. If some reduced cost is negative, then we are not at an optimal solution.

6. If the dual has multiple optimal solutions, then every primal optimal basic feasible solution is degenerate.

7. If the dual has multiple optimal solutions, then there exists a primal

optimal basic feasible solution which is degenerate.

8. The set of optimal solutions of a linear programming problem is bounded.

9. While solving an LP using the simplex method, if the bfs in the beginning of an iteration is degenerate, the objective value is the same after the iteration.

10. If $\mathbf{x}$ is primal feasible and $\mathbf{p}$ is dual feasible, then $\mathbf{p}'\mathbf{b} = \mathbf{c}'\mathbf{x}$.

11. If there is a nondegenerate optimal basis, then there is a unique optimal basis.

12. There exists a basis which is feasible for at least one of the primal and dual problems.

13. Suppose $\mathbf{B}$ is a primal feasible basis and $\mathbf{x}$ is the corresponding basic feasible solution. If $\mathbf{p}'\mathbf{b} = \mathbf{c}'\mathbf{x}$, then $\mathbf{p}$ is an optimal dual solution.

14. If $\mathbf{x}$ is an optimal solution, no more than m of its components can be positive. (Here, m is the number of rows of $\mathbf{A}$.)

PART II

1. (12 points)

(a) (5) Provide the definition of when two basic feasible solutions in a general polyhedron are *adjacent*.

(b) (7) Suppose that an iteration of the primal simplex method moves us from a basic feasible solution $\mathbf{x}$ to a different basic feasible solution $\mathbf{y}$. Briefly explain why $\mathbf{x}$ and $\mathbf{y}$ are adjacent, according to the definition of part (a).

2. (24 points)

Consider the following simplex tableau corresponding to a minimization problem in standard form under the usual assumptions.

*	0	0	2	γ
3	1	0	-2	-3
β	0	1	0	α

(a) (3) Suppose that $\beta > 0$. Find a necessary and sufficient condition for the current solution to be optimal.

(b) (6) Suppose that $\beta = 0$. Find a necessary and sufficient condition for the current solution to be optimal.

(c) (5) If the feasible set is known to be bounded, what does this imply about α?

(d) (5) Suppose that $beta = 1$ and that the current solution is optimal. Find necessary and sufficient conditions for the existence of other optimal solutions.

(e) (5) Suppose that $\beta = -1$. Find a sufficient condition for the problem to be infeasible.

3. (22 points)

Consider two nonempty polyhedra $P = \{\mathbf{x} \in \Re^{\mathbf{n}} \mid \mathbf{Ax} \leq \mathbf{b}\}$ and $Q = \{\mathbf{x} \in \Re^{\mathbf{n}} \mid \mathbf{Dx} \leq \mathbf{d}\}$. We are interested in finding out whether the two polyhedra have a point in common.

(a) (8) Devise a linear programming problem such that: if $P \cap Q$ is nonempty, it has an optimal solution and every optimal solution a point in $P \cap Q$; if $P \cap Q$ is empty, the problem is infeasible.

(b) (14) Suppose that $P \cap Q$ is empty. Use the dual of the problem you have constructed in part (a) to show that there exists a vector $\mathbf{c}$ such that $\mathbf{c}'\mathbf{x} < \mathbf{c}'\mathbf{y}$ for all $\mathbf{x} \in \mathbf{P}$ and $\mathbf{y} \in \mathbf{Q}$.