

6.251/15.081J Recitation 10

Week 10

1 Examples

Example 1.1. [1], exercise 7.31.

2 Review Problems

Example 2.1. *Projections of Polyhedral Cones.*

Given a polyhedral cone $C \subseteq \mathbb{R}^{n+p}$, prove that the projection of C along its first n components, $\Pi_n(C)$, is also a polyhedral cone. Does C being pointed imply $\Pi_n(C)$ is also pointed?

Example 2.2. [1], exercise 5.8.

Example 2.3. *Elimination.*

(Adapted from [2], ch. 3.) Consider a set of teams playing matches against each other, and assume each team either wins or loses a match in which it competes. We say a team is *eliminated* if, no matter what the results of the remaining matches are, they cannot finish with the most wins. In this problem we wish to prove an equivalent condition for a team to be eliminated.

We introduce the following notation. Let T be the set of teams other than the one whose elimination status is in question. Let $w(A)$ be the total number of current wins for a subset of teams $A \subseteq T$. Finally, let r_{ij} denote the number of games remaining between any two teams $\{i, j\} \subseteq T$, and let M denote the maximum number of possible wins for the team in question.

(a) Prove that if there exists a subset of teams $A \subseteq T$ satisfying

$$w(A) + \sum_{\{i,j\} \subseteq A} r_{ij} > M|A|, \quad (1)$$

then the team in question is eliminated. (*Hint*: You need no concepts from the class to argue this.)

- (b) Prove the converse of (a). In particular, this shows that 1 is an equivalent condition to the team being eliminated. (*Hint*: Formulate an appropriate maximum flow problem, then invoke the max-flow min-cut theorem.)
- (c) For the following team data, construct the network flow problem from (b) to determine whether the Algorithms are eliminated or not.

Team	Wins	To Play	D.	A.	V.	E.	B.
Degenerates	33	8	-	1	6	1	0
Algorithms	29	4	1	-	0	3	0
Vertices	28	7	6	0	-	1	0
Ellipsoids	27	7	1	3	1	-	2
Big- <i>O</i> 's	23	2	0	0	0	2	-

References

- [1] Bertsimas, D.; Tsitsiklis, J.N. *Introduction to Linear Optimization*. Athena Scientific, 1997.
- [2] Cook, W.J.; Cunningham, W.H.; Pulleyblank, W.R.; Schrijver, A. *Combinatorial Optimization*. John Wiley & Sons, Inc., 1998.