MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Fall 1999 — 6.251/15.081

Quiz 2

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1. (20 points)
Consider the problem

$$
\begin{array}{llllllll}
\text{minimize} & -5x_1 & - & x_2 & + & 12x_3 & & \\
\text{subject to} & 3x_1 & + & 2x_2 & + & x_3 & & = 10 \\
 & 5x_1 & + & 3x_2 & & & + x_4 & = 16 \\
 & x_1, \ldots, x_4 \geq 0. & & & & & &
\end{array}
$$

An optimal solution to this problem is given by $\mathbf{x} = (2, 2, 0, 0)$ and the corresponding simplex tableau is given by

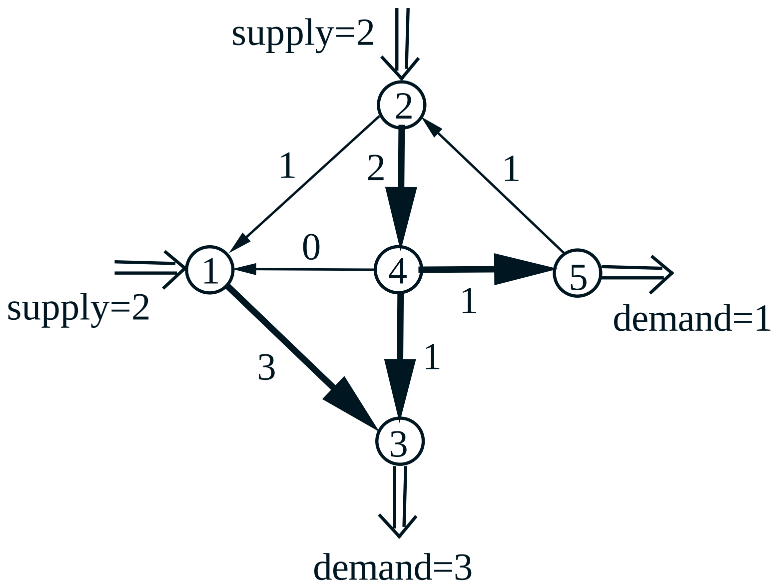
		x_1	x_2	x_3	x_4
	12	0	0	2	7
$x_1 =$	2	1	0	-3	2
$x_2 =$	2	0	1	5	-3

(a) (10) What is the range of values of c_1 for which $\mathbf{x} = (2, 2, 0, 0)$ is still optimal? (Assuming that all other coefficients remain constant.)

(b) (10) What is the range of values of b_1 for which $\mathbf{x} = (2, 2, 0, 0)$ is still optimal? (Assuming that all other coefficients remain constant.)

2. (35 points)

Consider the **uncapacitated** network flow problem shown below. The numbers next to each arc represent arc costs. Consider the tree indicated by thicker lines, and the corresponding basic solution.



(a) (5) What are the values of the arc flows corresponding to this basic solution?

(b) (6) Find the value of the reduced cost of each nonbasic arc, and explain why we have an optimal solution.

(c) (6) Find an optimal solution to the dual.

(d) (9) By how much can we decrease c_{52} and still maintain optimality? What is the optimal cost if c_{52} is decreased further?

(e) (9) What is the optimal cost (as a function of δ) if we change the supplies and demands to:

$$b_1 = b_2 = 2 + \delta, \qquad b_3 = -3 \qquad b_5 = -1 - 2\delta$$

What is the largest value of δ (call it δ^*) for which the same basis remains optimal? What happens if we try to increase δ beyond δ^*?

3. (22 points)
Consider the linear programming problem of minimizing $\mathbf{c}'\mathbf{x}$ subject to $\mathbf{Ax} \leq \mathbf{b}$. Show that the set of all vectors $\mathbf{c}$ for which the optimal cost is finite is a polyhedron.

4. (23 points)
Consider a polyhedron in standard form and assume that all basic feasible solutions are nondegenerate. Let the dimensions of the $\mathbf{A}$ matrix be $n \times m$. Prove that any feasible solution has at least m (strictly) positive components.