

6.251/15.081J Recitation 1

Week 1

1 Review

1.1 Basic Linear Algebra

1.1.1 Key Terms and Ideas

Know the formal definitions and have intuition for the following:

1. Linear independence
2. Subspace
3. Span
4. Basis
5. Dimension
6. Matrix range
7. Matrix nullspace
8. Matrix rank
9. Affine subspace

1.1.2 Commonly-Used Inversion and Transposition Properties

2 Examples

Example 2.1. (Taken from [1], chapter 3). Give an explicit solution of each of the following LPs, describing the conditions under which the problem is

unbounded below, infeasible, etc.

(a) *Minimizing a linear function over an affine set.*

$$\begin{aligned} & \text{minimize} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{A}\mathbf{x} = \mathbf{b}. \end{aligned}$$

(b) *Minimizing a linear function over a halfspace.*

$$\begin{aligned} & \text{minimize} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{a}^T \mathbf{x} \leq b, \end{aligned}$$

where $\mathbf{a} \neq \mathbf{0}$.

(c) *Minimizing a linear function over a rectangle.*

$$\begin{aligned} & \text{minimize} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{l} \leq \mathbf{x} \leq \mathbf{u}, \end{aligned}$$

where $\mathbf{l} \leq \mathbf{u}$.

(d) *Minimizing a linear function over the standard simplex.*

$$\begin{aligned} & \text{minimize} && \mathbf{c}^T \mathbf{x} \\ & \text{subject to} && \mathbf{1}^T \mathbf{x} = 1 \\ & && \mathbf{x} \geq \mathbf{0}. \end{aligned}$$

Example 2.2. (Taken from [1], chapter 3). The infinity (induced) norm of a matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, denoted $\|\mathbf{A}\|_{\infty, i}$, is defined as

$$\|\mathbf{A}\|_{\infty, i} = \max_{i=1, \dots, m} \sum_{j=1}^n |a_{ij}|.$$

This norm is sometimes called the max-row-sum norm.

Consider the problem of approximating a matrix, in the max-row-sum norm, by a linear combination of other matrices. That is, we are given $k+1$ matrices $\mathbf{A}_0, \dots, \mathbf{A}_k \in \mathbb{R}^{m \times n}$, and need to find $\mathbf{x} \in \mathbb{R}^k$ that minimizes

$$\|\mathbf{A}_0 + x_1 \mathbf{A}_1 + \dots + x_k \mathbf{A}_k\|_{\infty, i}$$

Express this problem as an LP.

References

- [1] Boyd, S., Vandenberghe, L. *Convex Optimization*. Course reader. Stanford University, 2001.