Lecture: Revealed Preference and Consumer Welfare

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1 Revealed Preference and Consumer Welfare

- We have so-far been applying an axiomatic approach to characterizing consumer choice (based on the five axioms given in the 1st lectures).

- But there is an approach to assessing utility that requires even fewer assumptions and nevertheless gives strong results. This approach is called *Revealed Preference.*

- Consider Figure 8#1 where the consumer sequentially faces two budget sets, $I_1 - I_1$ and $I_2 - I_2$. Point A on $I_1 - I_1$ is initially chosen. This point is said to be "revealed preferred" to all other feasible points inside of the budget set.

8#1
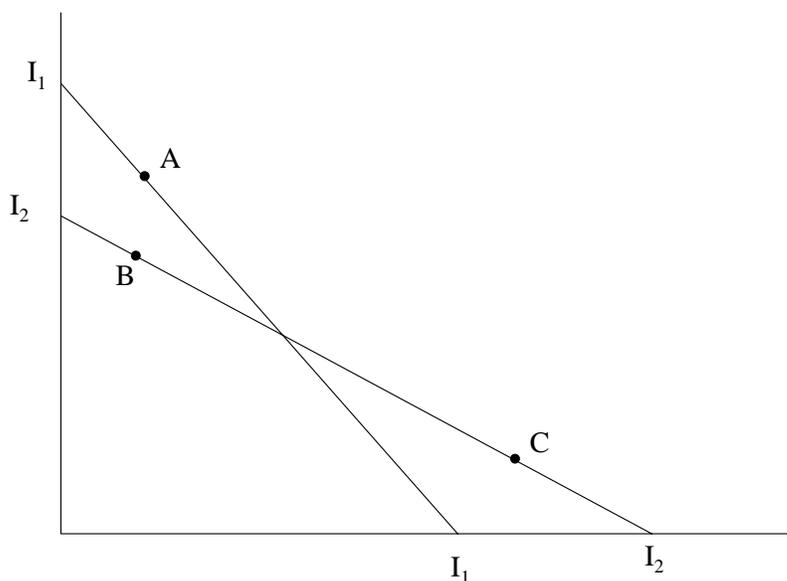
- Now the consumer is faced with $I_2 - I_2$.

  - If the consumer chooses point B, are they better or worse off? *Worse off*, because point A was revealed preferred to point B under the initial choice conditions.

1

– If they choose point C, are they better or worse off? The axiomatic approach would tend to suggest they are better off because unless the indifference curve tangent to point A has an extremely shallow slope, point C would probably put them on a higher indifference curve.

– But under Revealed Preference, the answer is ambiguous. The reason is that we do not have any revealed preference information on whether A is preferred to C or vice versa–these choices were never available simultaneously.

• Take a second example (See Figure 8#2).

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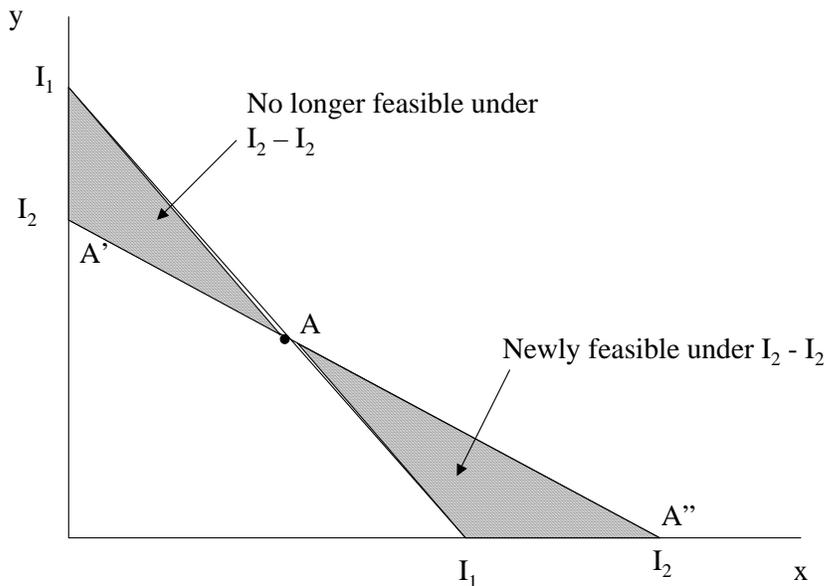
Here the second budget set rotates through the originally chosen point A on the first budget set. We do not know the consumer's new choice. Is the consumer better off, worse off, or can't we say?

– We know that point A r.p. $(A, A']$

– We do not know–but it is possible–that a point on $(A, A'']$ is preferred to A.

– We say that the consumer is "weakly" better off.

Definition 1 *Weak Axiom of Revealed Preference: If A, B feasible and A chosen, then at any prices and income where A, B are feasible, the consumer will choose A over B.*

This axiom says two things:

1. People choose what they prefer.

2. Preferences are *consistent*. Therefore, a single observed choice reveals a stable preference.

There is also a stronger form of this axiom.

Definition 2 *Strong Axiom of Revealed Preference (SARP): If commodity bundle 0 is revealed preferred to bundle 1 and bundle 1 is r.p. to bundle 2 and bundle 2 is r.p. to bundle 3... and bundle $k-1$ is r.p. to bundle k, then bundle k cannot be r.p. to bundle 0.*

SARP is simply WARP with an added transitivity assumption. But this places much greater strictures on behavior.

1.1 The power of WARP

- The result that $\frac{\partial X}{\partial P_x}|_{u=u_0} < 0$ (i.e., the compensated demand curve is always downward sloping) depends on an untested assumption about diminishing MRS, giving rise to indifference curves that are bowed inward towards the origin.

- Can we obtain the same result using only WARP?

- Suppose two points C, D on intersecting budget sets.

- Assume that these points are indifferent for consumer utility (see Figure 8#3). That is, the consumer has told us that he or she is indifferent.

$$C = (X_c, Y_c) \sim D = (X_d, Y_d).$$

8#3
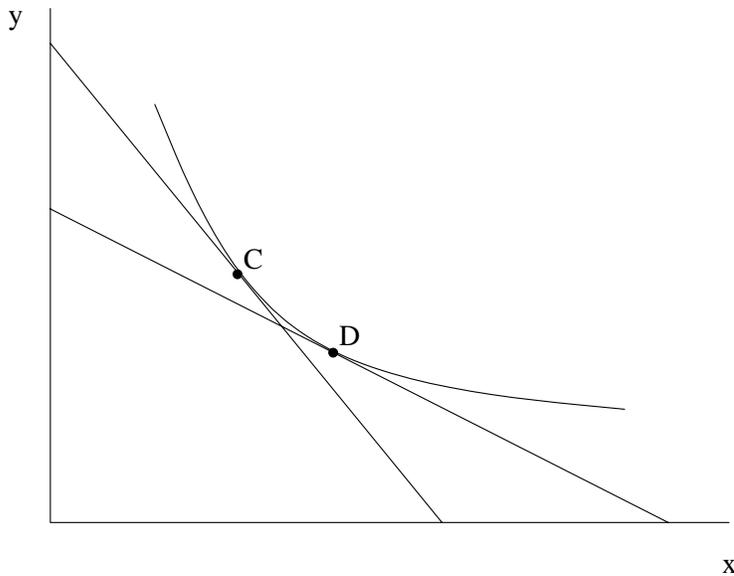
- Note that the 'indifference curve' drawn here is simply meant to represent the fact that the consumer *says* she is indifferent. There is *no notion of indifference curves* under the Revealed Preference approach.

- Since C, D do *not* lie in the same budget sets, when C is available, D is not and vice versa.

- Since $C \sim D$, is must be true by WARP that

$$P_x^c X_c + P_y^c Y_c \leq P_x^c X_d + P_y^c Y_d \text{ when } C \text{ is chosen,}$$
$$P_x^d X_d + P_y^d Y_d \leq P_x^d X_c + P_y^d Y_c \text{ when } D \text{ is chosen.}$$

- Rearranging, we get:

$$P_x^c(X_c - X_d) + P_y^c(Y_c - Y_d) \leq 0,$$
$$P_x^d(X_d - X_c) + P_y^d(Y_d - Y_c) \leq 0.$$

which simply says that at prices where C is purchased, D must have been at least as expensive as C (or C would have been purchased), and at prices where D is purchased, C must have been at least as expensive as D (or D would have been purchased).

- Combining, we get

$$(P_x^c - P_x^d)(X_c - X_d) + (P_y^c - P_y^d)(Y_c - Y_d) \leq 0. \tag{1}$$

- Now, consider a case where only the price of X, (P_x) changes and assume that $P_y^c = P_y^d$. Using (1), this gives

$$(P_x^c - P_x^d)(X_c - X_d) \leq 0,$$

which in calculus terms is equivalent to:

$$\frac{\partial X}{\partial P_x}\Big|_{u=u_0} \leq 0.$$

(Remember that $C \sim D$, so we are holding utility constant.)

- *So, WARP is sufficient to establish weakly downward sloping compensated demand curves. [Why Compensated? Because utility is held constant in this example since $C \sim D$.]*

- The *entire* idea of revealed preference is simply by using the weak notion of "choosing what you prefer," you get strong rationality properties, including:

  - Weakly downward sloping demand curves.

  - Only relative prices matter (as can be seen in the example above).

- We therefore don't have to make strong assumptions about diminishing MRS to get strong predictions about "rational" behavior.

2 Using WARP to evaluate the consequences of taxation

- For many reasons, governments need to tax:

    - Pay for public goods: Defense, law enforcement, regulatory agencies.

    - Transfer income–social insurance.

    - Correct externalities (pollution, 'sins').

- Are there better and worse ways to tax?

- Let's compare two types of taxes:

    - A lump-sum tax: reduces the consumer's budget by L.

    - A sales tax on a single good: charge tax t on X so that $P_x^t = P_x + t$.

- Obviously, consumer's are worse off for being taxed, but can we say anything stronger than that?

- See Figure 8#4.
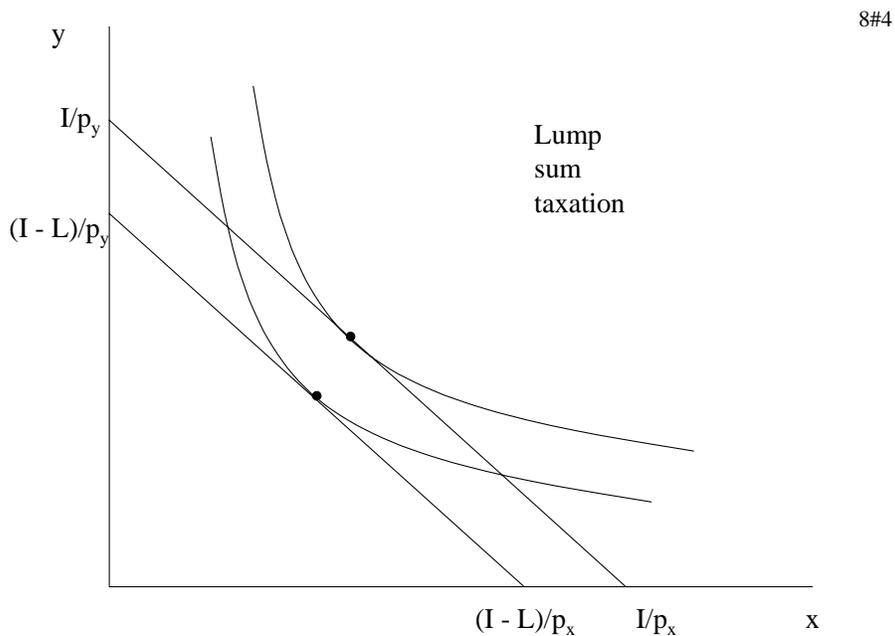
- Note the algebra of the lump sum tax:

$$XP_x + YP_y \;=\; I, \tag{2}$$

$$X_L P_x + Y_L P_y \;=\; I - L, \tag{3}$$

$$(X - X_L)P_x + (Y - Y_L)P_y \;=\; L$$

5

- To compare the lump-sum tax with a revenue equivalent sales tax, we need "revenue equivalence" (i.e., same level of taxes collected).

- Let's consider a tax of t^* on purchases of good X. So, for every X consumed, the consumer pays t^* in taxes.

- For t^* to be revenue-equivalent to L, the following condition must hold:

$$t^* \cdot d_x(P_x + t^*, P_y, I) = L.$$

In words, the "revenue equivalent" sales tax generates the same total taxation as L by charging t^* on each X purchased.

- To see this, note:

$$
\begin{aligned}
X_t(P_x + t^*) + Y_t P_y &= I, & (4) \\
I - X_t P_x - Y P_y &= X_t t^*.
\end{aligned}
$$

- So, for revenue equivalence, we need $X_t t = L$.

- We know from 2 that the budget set that characterizes the lump-sum tax is given by $X_L P_x + Y_L P_y = I - L$.

- So, the revenue equivalent tax *must* also put the consumer on this budget set. Hence,

$$I - X_t P_x - Y P_y = X_L P_x + Y_L P_y.$$

- Graphically, the revenue equivalent tax is the tax that causes the consumer to consume on the Lump-sum budget set. See Figure 8#5.

8#5

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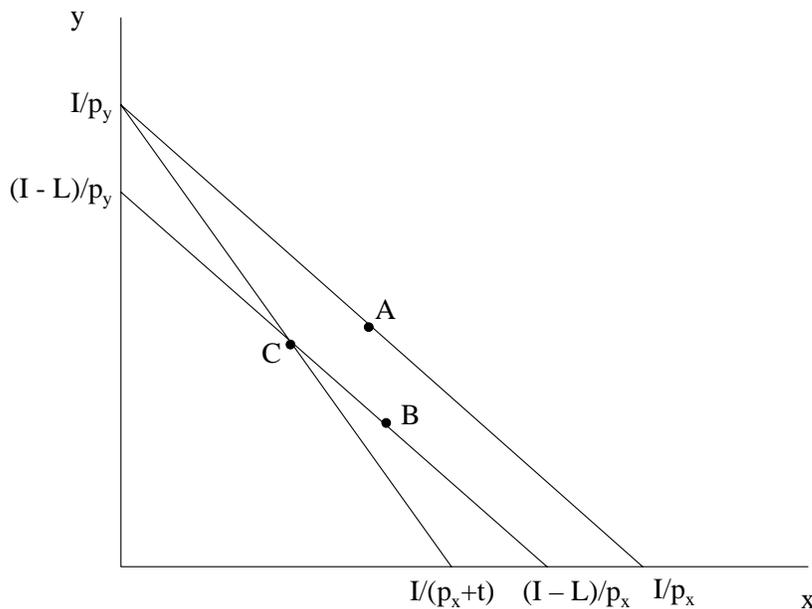
- Notice that the exact tax t^* that solves this problem depends upon consumer preferences. If consumption of X is highly elastic to the tax (that is, it falls precipitously), we'll need to set t fairly high to get $t^* X_t = L$. more

- Q: Since the tax puts the consumer back on the lump-sum budget set, does this imply that she is just as well off under either tax scheme?

- A: No, precisely the opposite. The consumer is strictly worse off under the sales tax than the lump-sum tax.

- Why: Because by shifting the price ratio, the tax has caused the consumer to choose a point on the lump-sum budget set that is not the most preferred point on this set. *Tax has distorted the choice.*

- This is a Revealed Preference argument: We know by WARP that B is the most preferred point on the budget set $[(I - L)/p_y, (I - L)/p_x]$. So, if taxation causes the consumer to choose any point other than B on this budget set, the consumer *must be* at least weakly worse off.

- A powerful and general result: If you must tax, you harm consumers less by simply taking a chunk of their budget than by distorting prices *and* ultimately taxing an identical share of their budget.

- Q: Drawing on the axiomatic approach to consumer utility, what is the exact distortion? In the lump-sum case, it remains true that

$$\frac{U_x}{U_y} = \frac{P_x}{P_y}.$$

- Whereas in the revenue equivalent taxation case, the consumer's 'optimal choice' will satisfy

$$\frac{U_x}{U_y} = \frac{P_x + t}{P_y}.$$

- Hence, they will under-consume X and overconsume Y. Their consumption choices do not reflect the 'real' cost of goods provided in the market – they are distorted by the tax.

- Question: What if the same proportional tax were applied to all goods?

2.1 Proof of distortionary impact of non-neutral taxation of goods

- Consider (implausibly) a tax that is *fully rebated* to the consumer:

$$t d_x (P_x + t, P_y, I + Z) = Z. \tag{5}$$

- This tax is revenue neutral for consumer; rebated exactly the amount paid in taxes (Z).

- Hence, only effect is to alter the price ratio faced by consumer.

- A critical (but strange) assumption here is that the consumer does not realize that the tax is fully rebated; that is, when the consumer buys X, she does not consider that she'll get $Z = tX$ tax rebate in return. If she did realize this, it would make the exercise pointless: the consumer would, in effect, not face a real tax on X.

- You can check that the consumer spends the original budget I by writing:

$$(P_x + t) \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I + Z.$$

- Subtracting (5) from both sides, we get

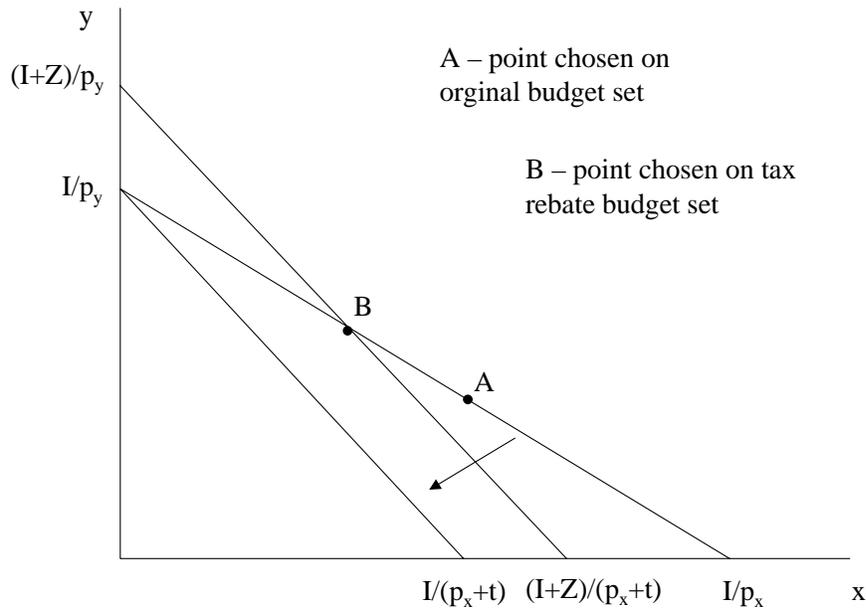$$P_x \cdot d_x(P_x + t, P_y, I + Z) + P_y \cdot d_y(P_x + t, P_y, I + Z) = I.$$

  Hence, the consumer is on the original budget set.

- But as long as the consumer changes the consumption bundle in response to the tax-ratio (i.e., as would occur for any utility function satisfying the standard 5 axioms), then the consumption bundle is 'distorted' by the tax:

$$
\begin{aligned}
d_x(P_x + t, P_y, I + Z) &\neq d_x(P_x, P_y, I), \\
d_y(P_x + t, P_y, I + Z) &\neq d_y(P_x, P_y, I).
\end{aligned}
$$

- In words, the consumer will be consuming on a different point on the original budget set I under the 'taxed' price ratio.

- If so, the consumer is *worse off* by Revealed Preference.

- Hence, the distortion induced by non-neutral taxation is that it causes the consumer to pick a non-preferred point on the 'true' (non-distorted) budget set.

- Note that this argument does not depend on any axioms of utility theory *other than* WARP. The essential point is:

  - We know that the consumer will *have to* pick a point on the original budget set for the tax to be revenue equivalent.

  - But rather than allow the consumer to simply face that budget set and choose the preferred point, we are distorting their behavior by shifting the slope while ultimately placing them back somewhere on the same line.

  - If they choose any other point than the preferred point on the undistorted budget set, they are weakly worse off. (Only weakly because we have no way of knowing whether the consumer was indifferent among multiple points on the original budget set, such as A and B).

y

$(I+Z)/p_y$

I/p_y

A – point chosen on
orginal budget set

B – point chosen on tax
rebate budget set

B

A

$I/(p_x+t)$ $(I+Z)/(p_x+t)$ I/p_x x

3   An Application: Lump Sum Taxation in New Hampshire [Byron Lutz, 2004]

Public schools in the United States are remarkably decentralized. There are over 14,000 school districts in the U.S.. Each may potentially set curriculum, choose testing policies, and raise revenue through local taxation.

Because of this decentralization, it is not surprising that there are vast disparities in per-pupil expenditures across U.S. states and among school districts within a state. What are the sources of these disparities? Two obvious candidates from consumer theory are:

- Differences in wealth across districts. If education is a normal good ($\partial E/\partial I > 0$), we would generally expect citizens in wealthy districts to want to tax themselves more to pay for schooling than citizens in less wealthy districts.

- Differences in tastes across districts. It's quite likely that there is underlying heterogeneity among U.S. citizens in their 'demand' for education – that is, how much education they would want to buy at given prices and income. So, even absent income effects, we'd expect there to be cross-district differences in expenditures on education.

These underlying taste differences are likely to be exacerbated by what Economists call *'Tiebout sorting.'* Citizens will sort themselves into communities in part based on the amenities provided. And the quality (and expense) of schools is an important amenity. Parents who have high demand for education will want to choose school districts that have good schools and, presumably, high local taxes (generally in the form of higher property taxes). And, similarly, parents with low demand for education will want to pool in communities with low taxes

and presumably weaker schools. This heterogeneity in tastes among individuals will be amplified by sorting among communities.

[Q: Why don't we have 'Tiebout sorting' into communities based on preferences for Coca Cola or sushi?]

3.1 The case of New Hampshire

Even by U.S. standards, New Hampshire (NH) is highly decentralized. Public goods in NH are extremely localized, and financed exclusively by property taxes. NH is the only state never to have enacted a state-wide income or sales tax and (as a consequence) has the nation's highest property taxes as a share of state income.

The state is also unique in heavy reliance on direct democracy for local decision making. Many school districts make decisions using the traditional open-meeting form of government. Instead of a legislature, citizens vote as a group at town meetings on proposed budgets and other fiscal decisions. From an economic perspective, there is an unusually strong presumption that in NH, local public goods expenditures reflect the preferences of local citizens.

Prior to 1999, 87 percent of total primary and secondary education revenue in NH came from local funding (the highest rate in the nation). This gave rise to particularly dramatic disparities in per-district expenditures.

Sunapee (a wealthy lake-front town) spent $8,233 per pupil while nearby Claremont spent only $4,223 per pupil.

In the *Claremont II* ruling in 1997, the NH Supreme Court declared the local property tax system used for educational funding unconstitutional. The court concluded that funding was inadequate in property-poor towns and the distribution of tax burdens was inequitable.

After 2 years of rancor, the NH legislature passed in 1999 a funding plan that appeared to pass constitutional muster. This plan established a formula to determine the 'cost of adequate education' (CAE) for each municipality.

A new, statewide property tax was implemented to fund this mandate. This tax is collected locally and retained locally.

However, if a municipality's tax revenue exceeds its CAE, it sends its excess revenue to the state (it's a 'donor' town).

Similarly, municipalities whose taxes fall short of the CAE receive a grant to make up the difference.

(However, the bulk of funding for the education grants comes primarily from state-wide, non-property tax revenue.)

Key points:

1. The reform left control of school finances under local control. There were no stipulations about how much municipalities had to spend on education.

2. The statewide tax was 'infra-marginal.' Every municipality raised more in local taxes then required by the statewide property tax. So, the tax was never 'binding' in that no community was forced to tax itself

more than it 'wanted to.'

3. The tax reform has no price implications; the marginal cost of providing public education was unaffected.

4. The subsidies gained by 'recipient' communities are unrestricted. They are a pure wealth transfer.

5. Similar, for 'donor' communities, subsidies are a pure negative wealth shock.

6. The reform was large in fiscal magnitude. The net new funding, $276 million, was equal to 19 percent of total pre-reform education revenue.

3.2 What does theory predict?

The legislature presumably intended that the fiscal transfers among municipalities would reduce inequality in educational expenditures. The Supreme Court must have concurred since it gave its approval to the plan.

But what does theory predict?

For simplicity, let's assume that each resident in a community has the same preferences and income. So, we can draw a 'community indifference curve' that is identical to an individual indifference curve. [This is an oversimplification of most models of political decision making, but it's not necessarily a terrible first approximation.]

Assume also that communities differ in both their wealth and demand for education (that is, even with prices and income equated, communities would choose different levels of education).

We can draw an indifference map with education on the x-axis and all other goods on the y-axis.

What is the budget set? It's effectively the entire income of the community. That's because communities can in theory tax themselves at 100 percent of income if they desire to spend all of their income on education.

The fact that they do not do this indicates that they have finite demand for education and other public goods (not surprising!).

Communities will differ in their chosen expenditures on education (E_j^* for community j) due to wealth and taste differences.

Now, imagine that communities receives a lump-sum grant (or lump-sum tax), L_j, from the state financing plan.

How does L_j effect the trade-off between education and all other public goods – that is, the slope of the budget set?

How does L_j effect the height of the budget set – that is, the set of all available goods?

What does this suggest about the likely effect of the lump-sum transfers on educational expenditures?

Does it matter whether education is a normal or inferior good?

3.3   What happened?

The experimental framework here is a familiar one, so we will not formally develop the difference-and-difference setup.

Assume that the grant/tax was effectively randomly assigned to communities.

Under this assumption, we can contrast the change in per-pupil expenditures in communities receiving a grant relative to those receiving neither a grant nor a tax (and/or to those receiving a tax).

This contrast should give us the causal effect of the grant on educational spending.

Equally good, we can assess the effect of the grant/tax on the change in *per-pupil local revenue* by community.

This outcome is the converse of the change in per-pupil expenditures equivalent because, if the grant/tax is not spent on education (or other public goods), it is implicitly returned to taxpayers in the form of reduced local taxes.

If the entire grant/tax were *not* spent on public goods, per-pupil local revenue would fall (rise) dollar-for-dollar with the grant/tax.

See Lutz Tables 2, 3a, and 5.

What does WARP say about the NH policy relative to an alternative where we gave communities the lump-sum grant and also required that they raise their education expenditures to (at least) the Cost of Adequate Education?

Are there any counter-arguments about why WARP may not be the right measure for social welfare in this case?

Would there by any way for the court to achieve the educational expenditure equilization it desired without 'distorting' choice in local communities?