

14.03 Fall 2004

Problem Set 2

Professor: David Autor

Due Friday, October 1, 2004 by 5pm

**1 Indirect utility function and expenditure function (10 points)**

Let  $U = x^{\frac{1}{3}}y^{\frac{2}{3}}$  be the utility function where  $x$  and  $y$  are two goods. Denote  $p_x$  and  $p_y$  as respectively the prices of the two goods  $x$  and  $y$ , and where  $M$  as the income of the consumer.

1. Derive the indirect utility function  $V(p_x, p_y, M)$
2. Derive the expenditure function  $E(p_x, p_y, U)$
3. Let  $p_x = 2$ ,  $p_y = 3$ , and  $M = 200$ . Find the utility maximizing bundle at those prices and income.

**2 Utility maximization (10 points)**

A consumer has the following utility function:

$$U(x_1, x_2) = (x_1 + 2)^{\frac{1}{2}} x_2^{\frac{1}{2}}$$

find the optimal consumption of  $x_1$  and  $x_2$  given prices  $p_1 = 6$ ,  $p_2 = 1$  and income  $I = 10$ . [Hint: This problem does not have a standard ‘interior’ solution.]

### 3 In-kind and cash transfers (15 points)

A consumer has the following utility over childcare  $c$  and food  $f$

$$U(x, y) = c^{\frac{1}{5}} f^{\frac{4}{5}}$$

The price of childcare is  $p_c = 2$ , the price of food is  $p_f = 4$  and income is  $I = 20$

1. What is the consumer's demand for childcare and food?
2. Suppose the government gives the consumer an income subsidy of  $S = 10$ . How will the consumer allocate the subsidy in the consumption of goods  $c$  and  $f$ ? [That is, what are  $c^*$  and  $f^*$  given the subsidy.]
3. Suppose now the government decides to give an in-kind transfer to the consumer. The in-kind transfer takes the form of 4 hours of childcare and 0.5 unit of food. Assume that the transfer cannot be re-sold. Draw a carefully labeled graph where you show the pre- and after transfer budget constraint. On this graph indicate the optimal choice for a constrained consumer and the optimal choice for an unconstrained consumer facing the same in-kind transfer.
  - (a) What are the new consumption levels after the in-kind transfer is given? What is the level of utility attained by the consumer at this consumption level?
  - (b) What is the minimum expenditure level required to attain the same utility if the consumer were buying all goods on the market?
  - (c) What is the cash equivalent of the in-kind transfer? [That is, the cash transfer that the government could give to provide the same level of utility to the consumer as the in-kind transfer?]

### 4 Food stamps program (15 points)

You are asked to evaluate the efficiency of the food stamps program. You run an experiment on two otherwise identical groups of benefit recipients. In month 1, you measure the baseline expenditures of both groups. In month 2, you give cash to group  $G1$  and food stamps to group  $G2$  and again measure expenditures. The total value of the transfer is \$100 per recipient in cash or food stamps.

You observe that within group  $G1$  (the cash group), 50% of the recipients increase their food consumption by \$100 and 50% of the recipients increase food consumption by \$60 and consumption of all other goods by \$40. [You can also assume that you have a 2nd control group who did not receive either a cash or a stamp transfer in either month. You find that their consumption of food and all other goods is identical in month 1 and 2. So the ‘time effect’ for the experiment appears to be zero, and you can ignore it.]

You are tempted to conclude that 20% of the transfers to  $G2$  is wasted because food stamp recipients who would have spent the money on all other goods are forced to spend it on food instead. You therefore reason that the dead-weight loss of food stamps is \$20 per \$100. After you read the Whitmore article, you conclude your initial estimate was incorrect.

1. Explain why the reasoning above (i.e, that the DWL of food stamps is \$20 per \$100) is incorrect. Is \$20 an overestimate of the DWL or an underestimate or is it indeterminate [explain]?
2. Describe qualitatively what information you would need to provide a correct estimate of the true DWL of the food stamp program. Draw a set of diagrams that shows the budget set faced by food stamp recipients for food versus all other goods. Show the indifference curve for the 50% of recipients who would like to spend the full \$100 of stamps on food (‘unconstrained recipients.’). Show the indifference curve for the 50% of recipients who would like to spend only \$60 on food but instead are required to spend \$100 (‘constrained recipients’). Draw the compensated demand function for food for a hypothetical constrained and unconstrained food stamp recipient. Explain, perhaps using a diagram, how the DWL loss of the food stamp program depends, in part, on the steepness (elasticity) of the compensated demand curve.
3. You later find out that recipients in  $G2$  sell 40% of their food stamps on the black market for 85% their value. Describe qualitatively how you incorporate this information in your DWL calculations. What does this fact imply about the marginal utility of food consumption for the food stamp recipients that use the black market?
4. The government introduces an Electronic Benefit Transfer (EBT) system: food stamp recipients now receive electronic debit cards instead of stamps. Because

debit cards cannot be used without identification and cannot be resold without detection, the EBT system entirely eliminates fraud. Hence, it shuts down the black market. Is the introduction of EBT likely to affect the DWL of the food stamp program? If yes, will it raise or lower it. Explain. [Assume that recipients who did not previously use the black market are indifferent between stamps and EBT.]

## 5 Waldfogel and Christmas (15 points)

On page 1334, Waldfogel writes, “While the pattern of cash-giving across types of givers is consistent with expected utility, the level of noncash giving is, at first blush, puzzlingly high... Indeed, one may view the deadweight losses as measures of giver satisfaction.” This problem asks you to explain this statement using consumer theory.

Consider the utility of a gift-giver as composed of (a) the sheer pleasure of giving a noncash gift ( $N$ ) and (b) the cash value the recipient places on the gift ( $V$ ). Assume that the giver spends a fixed amount  $I$  on the gift and can either give all cash valued by the recipient at  $V = I$  or an in-kind gift that will be valued by the recipient at  $V = \alpha I$  where  $0 < \alpha < 1$ . Assume further that the giver receives utility from the recipient’s enjoyment of the gift at exactly the amount that the recipient values the gift. So a rough representation of the giver’s utility function might be  $U(N, V) \simeq N + V$ .

1. What is the gift-giver’s decision rule about whether to give a cash or noncash gift?
2. Show generally that this model makes the prediction that the deadweight loss to the gift recipient (assuming that a noncash gift is given) is a measure of giver satisfaction of noncash giving as Waldfogel argues.
3. In this scenario, will the deadweight loss that Waldfogel is speaking of be an exact estimate, an upper bound estimate, or a lower bound estimate of the giver’s satisfaction (in monetary terms)?
4. Observe in Table 3 of Waldfogel that ‘significant others’ (spouses, boyfriends, girlfriends) tend to give in-kind gifts while distant relatives are more likely to give cash. How would you expand the conceptual model above to explain this fact? Does your model explain the pattern of ‘yields’ in Table 3? (Hint: how might  $\alpha$  depend on the ‘closeness’ of the relative?)

5. Could you use this model to explain why some people give ‘gift certificates,’ which are the equivalent of food stamps for a specific store (e.g., Amazon) rather than a specific product (e.g., food)? Explain.

## 6 Rational choice (10 points)

In the first period,  $p_x = 2$  and  $p_y = 1$  and the consumer buys 11 units of  $x$  and 8 units of  $y$ . In the second period,  $p_x = 1$  and  $p_y = 2$  and the consumer buys 10 units of  $x$  and 10 units of  $y$ . Prove that this set of choices is not consistent with rational utility maximization of preferences that satisfies all 5 axioms of consumer theory.

## 7 Short questions (4 points each + 1 free bonus point)

For each of the following questions state whether it is true, false or uncertain and explain your answer. No points will be given without explanation.

1. The effect of an in-kind transfer on consumer welfare is indeterminate because a constraint can be beneficial if well chosen.
2. The Lagrange multiplier in the utility maximization problem gives the shadow price of additional goods.
3. A good is only likely to be Giffen if expenditures on it are initially large relative to income.
4. In response to widespread student malnutrition at MIT, President Vest establishes an in-kind food transfer program which gives each student two slices of pizza per day valued at \$1 each. Every day after eating his two free slices, Fred buys a third slice from the MIT truck, also at \$1 per slice. Fred would have been better off if President Vest had given him \$2 per day to spend on whatever he liked.
5. Orange juice sells for \$2 per gallon and gasoline sells for \$1 per gallon. Although we don’t know how to measure utility, we do know that if a consumer buys both goods, she receives twice as much utility from orange juice as from gasoline.
6. A consumer with convex, ‘well-behaved’ indifference curves is indifferent between two bundles of  $X$  and  $Y$ :  $(4, 1)$  and  $(2, 9)$ . She therefore prefers the bundle  $(3, 8)$  to either of the first two.