# 14.03 Fall 2004

Problem Set 4 Solutions

November 8, 2004

1. **Gains from International Trade**

   Claudia (C) and Jesse (J) live in the land of Carvel, where only ice cream scoops (I) and brownies (B) are consumed. Claudia *only* enjoys brownies and ice cream together: one brownie with one scoop of ice cream on top. Jesse also enjoys both separately.

   Their utility functions and initial endowments are the following:

   Claudia: $U_C = \min(I_C, B_C)$; $E_C^I = 4 \quad E_C^B = 6$

   Jesse: $U_J = 0.5 * \ln(I_J) + 0.5 * \ln(B_J)$; $E_J^I = 6 \quad E_J^B = 4$

   To simplify the analysis assume $P_B = 1$.

   (a) Draw the Edgeworth Box, locate the endowment point and draw indifference curves for both people.

   (b) Find Claudia's and Jesse's demand functions. (i.e. $I_C$,$B_C$, $I_J$,$B_J$ as functions of $p_I$).[Hint: what is the optimal relation between $I_C$ and $B_C$?]

   *Answer:* To find the demand functions we need to solve the maximization problems for Claudia and Jesse.

   For Claudia: Max $U_C = \min(I_C, B_C)$

   $$\text{st. } p_I I_C + B_C = p_I E_C^I + E_C^B = 4p_I + 6$$

   To maximize her utility Claudia will choose $I_C$=$B_C$.Replacing this condition in her budget set we have:

   $$I_C * (p_I + 1) = 4p_I + 6 \rightarrow I_C = \frac{4p_I + 6}{p_I + 1}$$

   $$B_C = I_C = \frac{4p_I + 6}{p_I + 1}$$

   For Jesse: Max $U_J = 0.5 * \ln(I_J) + 0.5 * \ln(B_J)$

$$\text{st. } p_I I_J + B_J = p_I E_J^I + E_J^B = 6p_I + 4$$

Given that Jesse has a Cobb-Douglas Utility function his demand functions are:

$$I_J = \frac{0.5 * (6p_I + 4)}{p_I} = \frac{3p_I + 2}{p_I}$$

$$B_J = 0.5 * (6p_I + 4) = 3p_I + 2$$

(c) Using your estimations in (*b*) and the market clearing conditions, find the market equilibrium price ($p_I^*$). Replace this value in the demand functions and find the equilibrium consumption bundles $I_C$,$B_C$, $I_J$,$B_J$. Represent the equilibrium in the Edgeworth box. What are the utility levels for Claudia and Jesse at this equilibrium?

*Answer:* From the market clearing condition for Ice cream:

$$10 = I_C, +I_J = \frac{4p_I + 6}{p_I + 1} + \frac{3p_I + 2}{p_I} \longrightarrow p_I^* = 1$$

Replacing $p_I^* = 1$ in the demand functions we find:

$$I_C = B_C = I_J = B_J = 5$$

$$U_C = 5, U_J = \frac{\ln(25)}{2} = 1.6094$$

(d) Now suppose that Claudia and Jesse are considering opening their economy to world trade. The world price of ice cream ($p_I^W$) is 2 and the world price of brownies ($p_B^W$) is 1. Using the demand functions calculated in (*b*) find Claudia's and Jesse's demands for brownies and ice cream at the world prices. Represent them in the Edgeworth box.

*Answer:* Demands at world prices:

$$I_C = B_C = \frac{14}{3};\ I_J = 4, B_J = 8$$

(e) Show that if redistribution is not possible, Jesse would agree to open to trade, but Claudia would not. If you were a central planner and only cared for the sum of their utilities would you open the economy?

*Answer:* Replacing these values in the utility functions we find that:

$$\begin{aligned} Uc^W &= \frac{14}{3} < 5 = Uc^A \\ U_J^W &= \frac{\ln(32)}{2} > \frac{\ln(25)}{2} = U_J^A \end{aligned}$$

The central planner would open the economy, given that:

$$Uc^W + U_J^W = 6.40 > 6.35 = Uc^A + U_J^A$$

(f) Now assume that redistribution is possible. Find all possible transfers from Jesse to Claudia that would make Claudia *exactly* indifferent between free trade and remaining under autarky. Assume Jesse makes one of these transfers. Does he benefit from free trade?

*Answer:*

We know that Claudia will agree if she is at least as well off under international trade as she is under autarky. Given that we want to maximize Jesse's utility, the transfer should garantee that Claudia a utility of 5. The most direct way of doing this is by transfering $\frac{1}{3}$ units of ice cream and $\frac{1}{3}$units of brownies from Jesse to Claudia. All other possible transfers should guarantee that the endowment point after transfers is on the line that goes through the point in the box where $I_C = B_C = 5$ and has a slope of $-2$. Formally:

$$All\ T^B, T^I \text{ such that } (6 + T^B) = 15 - 2 * (4 + T^I)$$

2. **International Trade and Growth (Based on Frankel and Romer)**

As in Frankel and Romer we want to estimate the causal effect of Trade on Economic growth. We have information at the country level for gdp per capita growth ($G$) and a measure of openness to trade ($F$). The measure, developed by Sachs and Warner (1995) takes a value of **one** if NONE of the following conditions are true:

– average tariff rates are 40% or higher on imports of intermediate and capital goods
– non-tariff barriers cover 40% or more of imports of intermediate and capital goods
– a black market exchange rate is depreciated by 20% or more relative to the official exchange rate
– a socialist economic system
– the state has a monopoly on major exports.

and a value of **zero** if at least one of them is true. We have data for 62 countries, 36 for which $F = 0$ [The variable was constructed for 1990. Many countries opened their borders in the 90's]. Examples of countries with $F = 1$ are the US, Thailand,Australia and Taiwan. Examples with $F = 0$ are Argentina, Vietnam, China, Slovenia.

(a) Suppose we only have this two variables, and we calculate the following:

$\sum_{F=1} G_i/26 = 3.268$

$\sum_{F=0} G_i/36 = 0.786$

Propose a possible estimator for the causal effect of free trade on growth using these sums. What is the value of your estimator? What are the potential problems with this estimator? Do you think your estimator will understate or overstate the causal effect of trade on growth?

*Answer:* Proposed estimator:

$$E(G_{ii}^{iF} \mid F = 1) - E(G_i^A \mid F = 0) \simeq 3.268 - 0.786 = 2.48$$

Interpretation: If a country opens to free trade it will increase its per capita gdp growth by 2.48 percentage points. This estimator only measures the causal effect under the assumption that growth in the free-trade countries (in expectation) would be equal to growth in closed countries if it was not because of their openess to trade. Potential problems: countries that practice free trade are probably different from countries that don't in many dimensions other than trade policy, for example, policies with respect to capital markets, labor markets, privatization, etc. that might also affect growth. .

(b) Following Frankel and Romer, we look for an instrumental variable that can help us improve the plausibility of the causal estimation.

i. What are the conditions we want the instrumental variable to satisfy (please use formal notation to state the necessary conditions)?

*Answer:* Denote $Z$ a dummy instrumental variable.

- $E(G_{ii}^{iF} \mid Z = 1) = E(G_i^F \mid Z = 0)$ In words, the only way through which the instrument affects the dependent variable is through its effect on free trade.
- $E(F_{ii}^{iNT} \mid Z = 1) = E(F^{NT} \mid Z = 0)$. In words, the instrument is as good as randomly assigned. If it was not because of the instrument, countries would be equal in expectation.

We find data on whether a country is landlocked or not. If a country has direct access to the sea $L = 0$, if it doesn't $L = 1$.There are 11 landlocked countries in our data.

ii. Do you think it is plausible that this variable satisfied the conditions you stated in ($i$)? Why or why not [or you can give reasons for and against]?

*Answer:* The instrument can be considered almost as good as randomly assigned. Of course, the formation of countries is not completely exogenous, but given that most of the countries were formed various centuries ago, we can assume that whatever determined the limits of a country back then does not influence economic growth today. The first assumption is harder to justify. Though it is clear that being landlocked makes trade more difficult by increasing transportation costs, it also may affect many other factors that influence growth, such as industry composition, political instability, etc.

(c) You average the trade index for the landlocked and non-landlocked countries and find:

$\sum_{L=1} F_i/11 = 0.181$

$\sum_{L=0} F_i/51 = 0.47$

i. What is the causal effect of being landlocked on free trade under the assumptions you outlined in $(b)$? Why do you think landlocked countries are less likely to be open to trade? Can you think of any reasons why this relationship might go in the opposite direction?

*Answer:* The causal effect is $\sum_{L=1} F_i/11 - \sum_{L=0} F_i/51 = 0.181 - 0.47 = -0.289$. This coefficient says that being a landlocked country reduces the probability of having free trade by aprox. 29%. As mentioned above we might expect the effect going in this direction because of higher transportation costs. We might expect this relationship to go in the opposite direction because generally countries that are landlocked have more neighboring countries to trade with.

ii. You calculate the following averages:

$\sum_{L=1} G_i/11 = 0.968$

$\sum_{L=0} G_i/51 = 2.012$

Propose and calculate an IV estimator for the causal effect of trade on growth. Interpret its sign and magnitude.

*Answer:* $\frac{E(G_i^i|L=1)-E(G_i|L=0)}{E(F_i^i|L=1)-E(F_i|L=0)} \simeq \frac{0.968-2.012}{0.181-0.47} = 3.612$. The estimator says that opening to international trade increases growth by 3.1612%, a sizeable number.

iii. Compare the magnitude of your IV estimate with your estimate in part $(a)$. Did you expect to find this result? Explain.

*Answer:* The IV estimator is larger. We would have expected it to be lower given that we think that our estimate in (a) is upwardly bias as countries open to trade should also have other policies that might spur growth.

3. **Short Questions**

(a) Consider the four choices:

**a**. $1,000,000 for sure

**b**. 10% chance of $5,000,000

89% chance of $1,000,000

1% chance of $0

**c.** 10% chance of $5,000,000

90% chance of $0

**d**. 11% chance of $1,000,000

89% chance of $0

Before reading further, choose which you would prefer between **a** and **b**, and then choose which you would prefer between **c** and **d**. [The choice you make does not affect your grade.] It is commonly observed that people prefer **a** to **b**, and prefer **c** to **d**. Show that this pair of choices is inconsistent with expected utility maximization.

*Answer:* People prefer A to B $\iff U(1000000) > 0.1 * U(5000000) + 0.89 * U(1000000) + 0.01 * U(0)$.

Adding $0.89 * U(0) - 0.89 * U(1000000)$ to both sides of the inequality gives us:

$U(1000000) - 0.89 * U(1000000) + 0.89 * U(0) > 0.1 * U(5000000) + 0.89 * U(1000000) + 0.01 * U(0) - 0.89 * U(1000000) + 0.89 * U(0)$

Rearranging terms,

$0.11 * U(1000000) + 0.89 * U(0) > 0.1 * U(5000000) + 0.9 * U(0) \iff U(D) > U(C)$

(b) Richie, an expected utility maximizer, places an even bet of $50,000 on the Red Sox winning the World Series. If he has a utility function that is logarithmic in wealth [e.g., $U = \ln(W)$] and his current wealth is $200,000, what is the *minimum* probability he must place on the Red Sox wining the Series? [That is, Richie must believe that the probability that the Sox win is greater than equal to $\rho$, a number you calculate.]

*Answer:*

The minimum probability $\rho$ is such that:

$$\rho * \ln(250000) + (1 - \rho) * \ln(150000) = \ln(200000) \rightarrow \rho = 0.563$$

(c) Angelo has a utility function of the form $U = .\ln(W)$ He is offered a lottery that gives $50 with a probability of $\frac{1}{2}$ and $10 with a probability of $\frac{1}{2}$. What is the maximum amount of money he is willing to pay to participate? (Assume his initial wealth is $30).

*Answer:* He would pay an amount ($P$) such that:

$$\begin{aligned} U(Participating) &= U(not\ participating) \iff \ln(30) = 0.5 * \ln(30 - P + 50) + 0.5 * \ln(30 - P + 10 \\ P &= 23.94 \end{aligned}$$

4. **Uncertainty and Insurance**

Forecasters predict there is a 50 percent probability that the upcoming growing season will be a drought. Assume that Farmer Jane is an expected utility maximizer with utility function $U(W) = ln(W)$. Her initial wealth is $0.

(a) Is Jane risk averse (yes/no)?

*Answer: Yes, her utility function is concave.* $U''(W) = -\frac{1}{W^2}$

Jane initially has the choice between two crops with payoffs:

| | Normal Rain | Drought |
|---|---|---|
| Potatoes | \$5,000 | \$40,000 |
| Strawberries | \$20,000 | \$12,000 |

b. If she can only plant one crop, which crop should she plant?

She should plant Strawberries because $U(Strawberries) = 0.5\ln(20,000)+0.5\ln(12,000) = 9.65 > 9.56 = 0.5\ln(5,000) + 0.5\ln(40,000) = U(Potatoes)$

c. Assume she can instead plant half her land with each crop. Which crop mix gives the highest expected income (all potatoes, all strawberries half of each). Which crop mix should Jane choose? Explain.

*Answer:*. If she plants half with each crop she would get:

$$0.5\ln(0.5 * 20000 + 0.5 * 5000) + 0.5\ln(0.5 * 12000 + 0.5 * 40000) = 9.79$$

which is greater than $U(Strawberries) = 9.65$.Therefore she should plant half and half.

d. Assume Jane can choose any combination of Potatoe and Strawberry crops, provided that their total sums to 100 percent. What mix of crops maximizes Jane's expected utility?

*Answer:* Define $\alpha$ the fraction of the land in which Jane plants strawberries. To find the optimal $\alpha$ we have to solve:

$Max_\alpha\ 0.5\ln(\alpha * 20000 + (1-\alpha) * 5000) + 0.5\ln(\alpha * 12000 + (1-\alpha) * 40000) \iff Max_\alpha(\alpha * 20 + (1-\alpha) * 5) * (\alpha * 12 + (1-\alpha) * 40)$

FOC:$15 * (\alpha * 12 + (1-\alpha) * 40) - 28 * (\alpha * 20 + (1-\alpha) * 5) = 0 \rightarrow \alpha = \frac{23}{42}$

e. Assume Jane decides to plant half her land with each crop. She is offered Strawberry insurance. This insurance costs \$5,000 and pays \$10,000 in the case of a drought. [Hence, the policy is actuarily fair.] Should Jane buy it? Explain your answer in light of your response to queston $(a)$.

$Answer$ : If she buys the insurance her utility is: $0.5\ln(0.5*5000+0.5*15000)+0.5\ln(0.5*40000 + 0.5 * 17000) = 9.72$ which is smaller than without insurance. Therefor she should NOT buy it.The reason is that, by planting potatoes, she is already insured in case of a draught. Given that the outcome for potatoes is very low in normal rain conditions (and there is no insurance for potatoes) she is better off having a high return to strawberries in this state of the world.

5. Bill is a Von-Neumann Morgenstern expected utility maximizer with a well-behaved, continuously differentiable utility function (i.e., no kinks or inflection points). Bill is presented with the following choices:

**a**. $\$1,000$ for sure
**b.** 50% chance of $800, 50% chance of $\$1,500$
**c.** $500 for sure
**d.** 50% chance of $400, 50% chance of $900

Bill is indifferent between **a** and **b** and is also indifferent between **c** and **d**. (Note: this does not imply that he is indifferent between **a** and **c** or **b** and **d**.)

(a) Is Bill risk neutral, risk averse, risk loving, or can't you tell? Explain.

*Answer:* He is risk averse because U(b)<E(b).

(b) He is now faced with the following choice:

**e.** $750 for sure
**f.** 25% chance of $400, 25% chance of $900, 25% chance of $800, 25% chance of $\$1,500$

Will Bill choose **e** or **f**, or is he indifferent between them, or is not possible to tell? (Prove your answer)

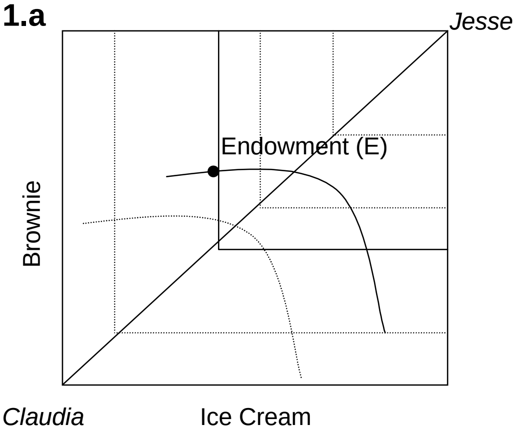
*Answer:* He will choose E over F.

Proof: $U(E) = U(750) >^{(1)} 0.5 * U(1000) + 0.5 * U(500) =^{(2)} 0.5 * (0.5 * U(800) + 0.5 * U(1500)) + 0.5 * (0.5 * U(400) + 0.5 * U(900)) = 0.25 * U(800) + 0.25 * U(1500) + 0.25 * U(400) + 0.25 * U(900) = U(F)$

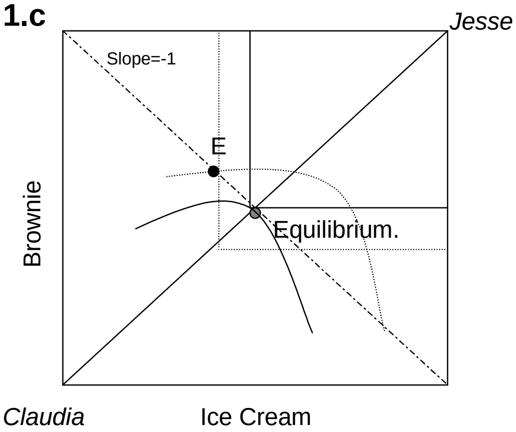
$^{(1)}$Bill is risk averse

$^{(2)}$Independence axiom.

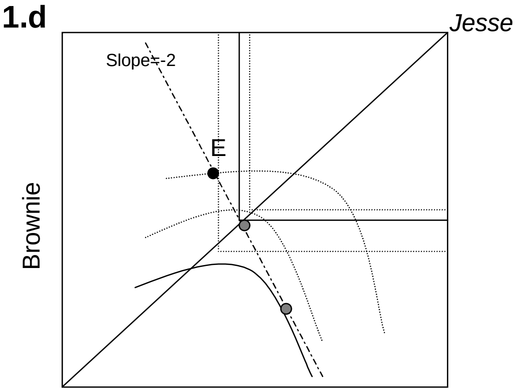
**1.a**


Jesse
Endowment (E)
Brownie
Claudia
Ice Cream

**1.c**


Jesse
Slope=-1
E
Equilibrium.
Brownie
Claudia
Ice Cream

**1.d**


Jesse
Slope=-2
E
Brownie