

The Expenditure Function: An Application to the Economics of Food Stamps

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1 The Expenditure Function

We are next going to look at a potentially richer (and better) application of consumer theory: the value of Food Stamps.

Before that, we need some more machinery.

So far, we've analyzed problems where income was held constant and prices changes. This gave us the Indirect Utility Function.

Now, we want to analyze problems where utility is held constant and expenditures change. This gives us the Expenditure Function.

These two problems are closely related – in fact, they are ‘*duals*.’

Most economic problems have a *dual problem*, which means an inverse problem.

For example, the dual of choosing output in order to maximize profits is minimizing costs at a given output level: cost minimization is the dual of profit maximization.

Similarly, the dual of maximizing utility subject to a budget constraint is minimizing expenditures subject to a utility constraint.

1.1 Setup of expenditure function

Consumer's problem: maximize utility subject to a budget constraint.

Dual: minimizing expenditure subject to a utility constraint (i.e. a level of utility you must achieve)

This dual problem yields the “expenditure function”: the minimum expenditure required to attain a given utility level.

Setup of the dual

1. Start with:

$$\begin{aligned} & \max U(x, y) \\ \text{s.t. } & p_x x + p_y y \leq I \end{aligned}$$

2. Solve for $x^*, y^* \Rightarrow v^* = U(x^*, y^*)$ given p_x, p_y, I .

$$V^* = V(p_x, p_y, I)$$

V is the indirect utility function.

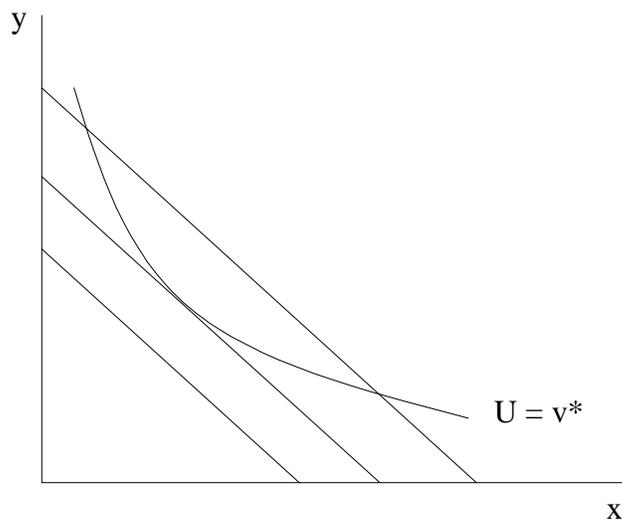
3. Now solve the following problem:

$$\begin{aligned} & \min p_x x + p_y y \\ \text{s.t. } & U(x, y) \geq v^* \end{aligned}$$

gives $E^* = p_x x^* + p_y y^*$ for $U(x^*, y^*) = v^*$.

$$E^* = E(p_x, p_y, V^*)$$

1.2 Graphical representation of dual problem



The dual problem consists in choosing the lowest budget set tangent to a given indifference curve.

Example:

$$\begin{aligned} \min E &= p_x x + p_y y \\ \text{s.t. } x^{.5} y^{.5} &\geq U_p \end{aligned}$$

where U_p comes from the primal problem.

$$L = p_x x + p_y y + \lambda (U_p - x^{.5} y^{.5})$$

$$\frac{\partial L}{\partial x} = p_x - \lambda \cdot 5x^{-.5} y^{.5} = 0$$

$$\frac{\partial L}{\partial y} = p_y - \lambda \cdot 5x^{.5} y^{-.5} = 0$$

$$\frac{\partial L}{\partial \lambda} = U_p - x^{.5} y^{.5} = 0$$

The first two of these equations simplify to:

$$x = \frac{p_y y}{p_x}$$

We substitute into the constraint $U_p = x^{.5}y^{.5}$ to get

$$\begin{aligned}U_p &= \left(\frac{p_y y}{p_x}\right)^{.5} y^{.5} \\x^* &= \left(\frac{p_y}{p_x}\right)^{.5} U_p, \quad y^* = \left(\frac{p_x}{p_y}\right)^{.5} U_p \\E^* &= p_x \left(\frac{p_y}{p_x}\right)^{.5} U_p + p_y \left(\frac{p_x}{p_y}\right)^{.5} U_p \\&= 2p_x^{.5} p_y^{.5} U_p\end{aligned}$$

1.3 Expenditure function: What is it good for?

The expenditure function is an essential tool for making consumer theory operational for public policy analysis.

Using the expenditure function, we can ‘monetize’ otherwise incommensurate tradeoffs to evaluate costs and benefits.

The need for this type of calculation arises frequently in public policy analysis, and is the basis for most cost-benefit analyses.

We’ve been stressing all semester that we don’t know that ‘utils’ are. That presents a problem if we want to determine *how much* harm or benefit a certain policy imposes on an individual.

The expenditure function gives us a convenient way to potentially circumvent this problem.

Though we don’t know how to measure utils, we do know that money increases utility (i.e., through the indirect utility function by relaxing the budget constraint).

Using the expenditure function, we can figure out *how much money* we would have to give or take away from the consumer to leave her equally well off after a policy is implemented. So, the expenditure function permits us to calculate a ‘money metric.’

Example: we might consider a policy that of banning Sport Utility Vehicles because they cause a disproportionate share of air pollution and increase oil dependence.

How much harm does this policy do to potential buyers of SUVs?

We can't answer this question in utils. We can potentially answer it by determining how much money we would need to give these buyers to leave them equally well off as before the ban. This calculation depends on the expenditure function.

Let's say that consumer utility of would be SUV-buyers prior to the ban is given by \bar{U} and expenditures by:

$$E_{pre} = E(p_a, p_{suv}, \bar{U})$$

To attain the same level of utility after the ban, would-be buyers would need this much :

$$E_{post} = E(p_a, p_{suv} = \infty, \bar{U}).$$

The difference $E_{post} - E_{pre}$ is the amount of money that we would need to compensate SUV buyers to leave their utility unaffected by the ban.

Of course, we don't usually *know* the expenditure function, so this isn't as easy in practice as it is in theory.

But it turns out that if we have an estimate of the compensated elasticity of demand for a good, this is often enough to make a rough calculation

You'll see this in the Whitmore article.

1.4 Relation between Expenditure function and Indirect Utility function

How do solutions to Dual and Primal problems compare?

Examine the relation between expenditure function and indirect utility function.

$$\begin{aligned}
V(p_x, p_y, I_0) &= U_0 \\
E(p_x, p_y, U_0) &= I_0 \\
V(p_x, p_y, E(p_x, p_y, U_0)) &= U_0 \\
E(p_x, p_y, V(p_x, p_y, I_0)) &= I_0
\end{aligned}$$

Expenditure function and Indirect Utility function are *inverses* one of the other.

Let's verify this in the example we saw above.

Recall that primal gave us factor demands x_p^* , y_p^* as a function of prices and income (not utility).

Dual gave us expenditures (budget requirement) as a function of utility and prices.

$$x_p^* = \frac{I}{2p_x}, y_p^* = \frac{I}{2p_y}, U^* = \left(\frac{I}{2p_x}\right)^{.5} \left(\frac{I}{2p_y}\right)^{.5}$$

Now plug these into expenditure function:

$$E^* = 2U_p p_x^{.5} p_y^{.5} = 2 \left(\frac{I}{2p_x}\right)^{.5} \left(\frac{I}{2p_y}\right)^{.5} p_x^{.5} p_y^{.5} = I$$

Finally notice that the multipliers are such that the multiplier in the dual problem is the inverse of the multiplier in the primal problem.

$$\begin{aligned}
\lambda_P &= \frac{U_x}{p_x} = \frac{U_y}{p_y} \\
\lambda_D &= \frac{p_x}{U_x} = \frac{p_y}{U_y}
\end{aligned}$$

1.5 Demand Functions

Now, let's use the Indirect Utility function and the Expenditure function to get Demand functions.

To now, we've been solving for:

- Utility as a function of prices and budget
- Expenditure as a function of prices and utility

Implicitly we have already found demand schedules.

A demand schedule is immediately implied by an individual utility function.

For any utility function, we can solve for the quantity demanded of each good as a function of its price with the price of all other goods held constant and *either* income held constant *or* utility held constant.

1.5.1 Marshallian demand ('Uncompensated' demand)

In our previous example where:

$$U(x, y) = x^{.5}y^{.5}$$

we derived:

$$\begin{aligned} x(p_x, p_y, I) &= .5 \frac{I}{p_x} \\ y(p_x, p_y, I) &= .5 \frac{I}{p_y} \end{aligned}$$

In general we will write these demand functions (for individuals) as:

$$\begin{aligned} x_1^* &= d_1(p_1, p_2, \dots, p_n, I) \\ x_2^* &= d_2(p_1, p_2, \dots, p_n, I) \\ &\dots \\ x_n^* &= d_n(p_1, p_2, \dots, p_n, I) \end{aligned}$$

We call this "Marshallian" demand after Alfred Marshall who first drew demand curves.

1.5.2 Hicksian demand ('Compensated' demand)

Similarly we derived that:

$$x(p_x, p_y, U) = \left(\frac{p_y}{p_x}\right)^{.5} U_p$$

$$y(p_x, p_y, U) = \left(\frac{p_x}{p_y}\right)^{.5} U_p$$

In general we will write these demand functions (for individual) as:

$$x_{1,c}^* = h_1(p_1, p_2, \dots, p_n, U)$$

$$x_{2,c}^* = h_2(p_1, p_2, \dots, p_n, U)$$

$$\dots$$

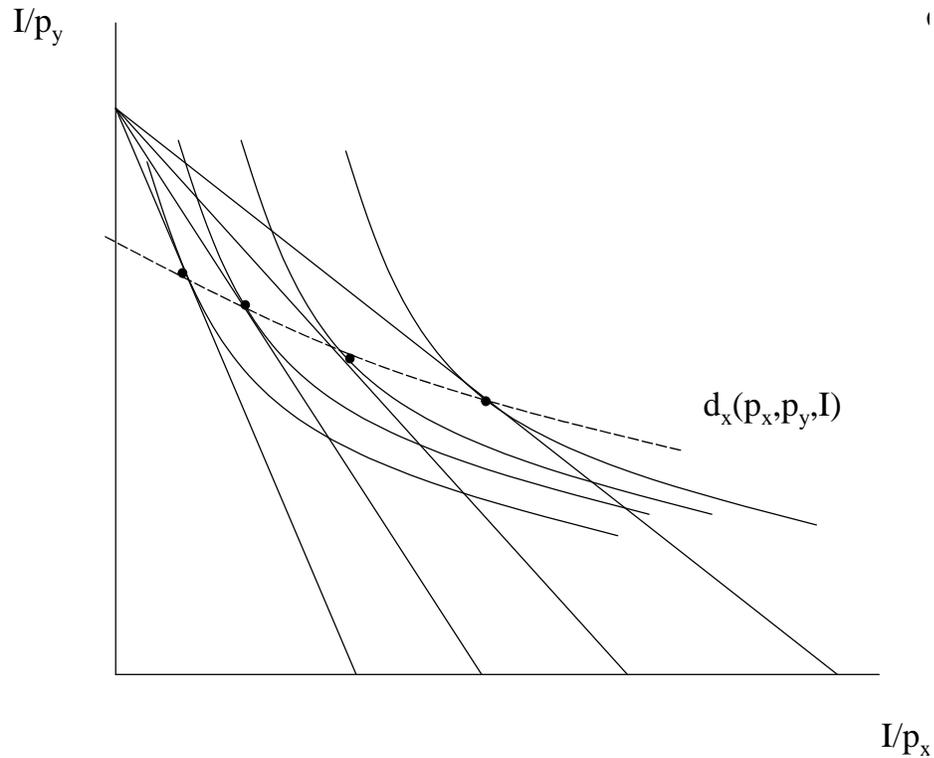
$$x_{n,c}^* = h_n(p_1, p_2, \dots, p_n, U)$$

This is called “Hicksian” or compensated demand after John Hicks.

This demand function takes utility as an argument, not income. This turns out to be an important distinction.

1.6 Graphical derivation of demand curves

A demand curve for x as a function of p_x



So a demand function is a set of tangency points between indifference curves and budget set holding I and p_y (all other prices) constant.

What type of demand curve is this?

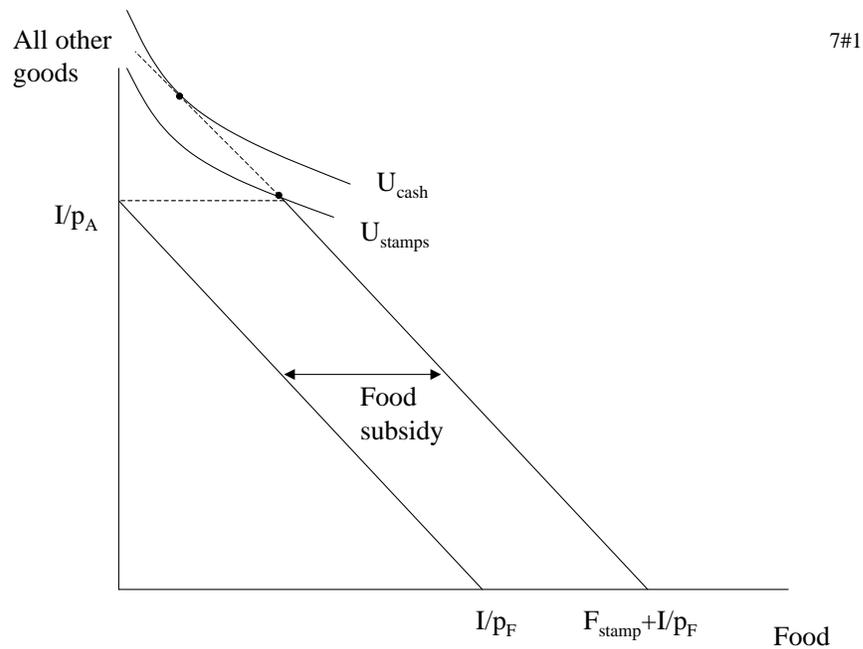
Marshallian ($d_x(p_x, p_y, I)$). Utility is not held constant, but income is.

Now, we have the tools to analyze the Food Stamp program.

2 In-Kind Transfers: An application of consumer theory

- As we discussed in a previous class, there are many ‘in-kind’ versus cash transfers made by government to citizens:
 - Food stamps.
 - Public housing.

- Child care assistance.
 - Medical care.
 - Schooling.
- These transfers have two economic effects:
 - They shift the budget set upward so that the consumer can potentially buy more of the subsidized good *and* all other goods.
 - They place a kink in the budget set at the subsidy level of the in-kind good—thus they ‘force’ consumption of at least the threshold level of the good.
 - See Figure 7#1



- What are the consequences of this type of transfer for consumer welfare?
 - For a consumer who consumes on non-kinked section of new budget set?
 - For a consumer who consumes at the kink?

- Can they make a consumer worse off?

2.1 Food-stamps

2.1.1 The U.S. Food Stamp Program: Some facts

- In 2002, the U.S. Department of Agriculture distributed \$19.1 billion dollars in food stamps and spent an additional \$2.4 billion dollars on administration.
- There were 8.2 million households receiving food stamps
- 19 million recipients
- Receiving on average \$79 per person per month, \$950 per person per year.

2.1.2 Advantages of in-kind transfers

1. Guarantee nutrition?
2. Prevent use of cash on drugs, alcohol cigarettes \iff Paternalism: invalid preferences
3. What is “valid” use of public money? Food versus recreation.
4. Political necessity (for public support).

2.1.3 Disadvantages of in-kind transfers

1. Restrict/distort choice. Economists think this is a bad.
2. Administration/enforcement costs. Estimated that **half** of the cost of food stamp administration is fraud prevention.
3. Who is made better off for enforcing this restriction? (Does it pass a Pareto test?)
4. Creation of underground market for trade in stamps (‘shadow market’).
5. Creation of criminals.

2.1.4 Note: other ‘shadow markets’

1. Human organ sales.
2. Donor eggs for infertile couples.
3. Adopted children.
4. Prostitution.
5. Illicit drugs.
 - General principle: When you prevent trades that people would otherwise engage in, market will attempt to undo this distortion through a ‘shadow market.’
 - The cost of enforcement may be high. Consider Prohibition.
 - Some otherwise law-abiding citizens will engage in crime, go to jail, and perhaps commit other ancillary antisocial activities (consider the violence of the illicit drug trade).

3 The value of food stamps: Policy questions

What are the key policy questions you would want to answer in comparing cash to in-kind food stamp transfers?

1. Are recipients “distorted?” That is, do they indeed spend more on food than they otherwise would without food stamps?
2. What share of food stamps are sold onto the black market? And at what price?
3. Does cash versus in-kind have any effect on nutrition?
4. How costly are cash versus in-kind programs to administer?

3.1 The value of food stamps: the Whitmore study.

- Analyzes a pair of food-stamp experiments in San Diego and Alabama implemented in the early 1990s by the U.S. Department of Agriculture.
- “Cash out” experiments: Food stamp benefits paid in cash to a random subset of recipients instead of food stamp coupons.
- Idea: Compare food and other expenditures among households receiving stamps and equivalent households randomly assigned cash instead.
- Notice: There is no pre-period (i.e., baseline data), so this is not a “difference-in-difference” comparison (unlike Card and Krueger or Jensen and Miller).
- Is that a problem? Not necessarily. If the randomization is valid and we have a reasonably sized sample, we can be fairly confident that the counterfactual outcomes for the treatment and control groups are comparable. In that case, we can compare outcomes in the post period to assess the counterfactual for either group.
- For concreteness, let $X = 1$ denote cash and $X = 0$ denote stamps. Let Y_{i0} equal food expenditures if assigned to stamps and Y_{i1} equal food expenditures if assigned to cash. If the randomization is valid, $E(Y_{i0}|X_i = 0) = E(Y_{i0}|X_i = 1)$ and $E(Y_{i1}|X_i = 0) = E(Y_{i1}|X_i = 1)$.
- These expressions say that if the treatment and control groups were switched prior to the experiment (treatment assigned to control and control assigned to treatment), average food spending for the *accidental* control group would be equal to the food spending of the *intended* control group had it not been swapped – and similarly for the treatment group. .

3.2 ‘Distorted’ versus ‘non-distorted’ households

- Would we expect all or even most households to be made worse off by food stamps? Answer: No. Low income families spend a considerable share of their income on food—*about 30 percent of the household budget in the Whitmore sample, which is twice what average Americans spend.*

- And food stamps are not that generous: \$111 - \$370 per month for households of 1 - 4 children at the time of the study. This is less than 1/3rd of the typical low-income household budget.
- How do you measure which households are ‘distorted?’
In a pre-post experiment, this would be easy. How? Look at households that decreased their food consumption after they were ‘cashed out.’ They were distorted by the program.

In Whitmore’s study, this is harder because there is no baseline data.

- Whitmore’s first approach is to label a household distorted if its monthly food spending is less than its food stamp amount.
For the cash-recipient households, this poses no problem.
For the check recipients, this means that they don’t spend all of their food stamps.
- By this definition, **18 to 21 percent** of households are distorted.
- What does consumer theory say about these households? By the axiom of non-satiation, they should not exist—unless they are selling stamps onto the black market.
- On the other hand, the USDA reports that \$500 million a year in food stamps (of \$22 billion) goes unclaimed. That’s about 5 percent of all stamps, and is potentially consistent with 20 percent of households that spend only 75 percent of their stamps.
- Whitmore uses a number of other definitions, but they lead to similar conclusions.
- SEE WHITMORE TABLES 2a and 2b. Using the difference in food spending among ‘distorted’ households that do and do not receive the cash grant, Whitmore calculates a rough measure of the ‘distorted share’ of food stamp benefits. This is simply the difference between what they

would have spent on food and what they did spend on food divided by what they would have spent:

$$DS = \frac{F_s - F_c}{F_c},$$

where c stands for cash transfer and s is for stamps.

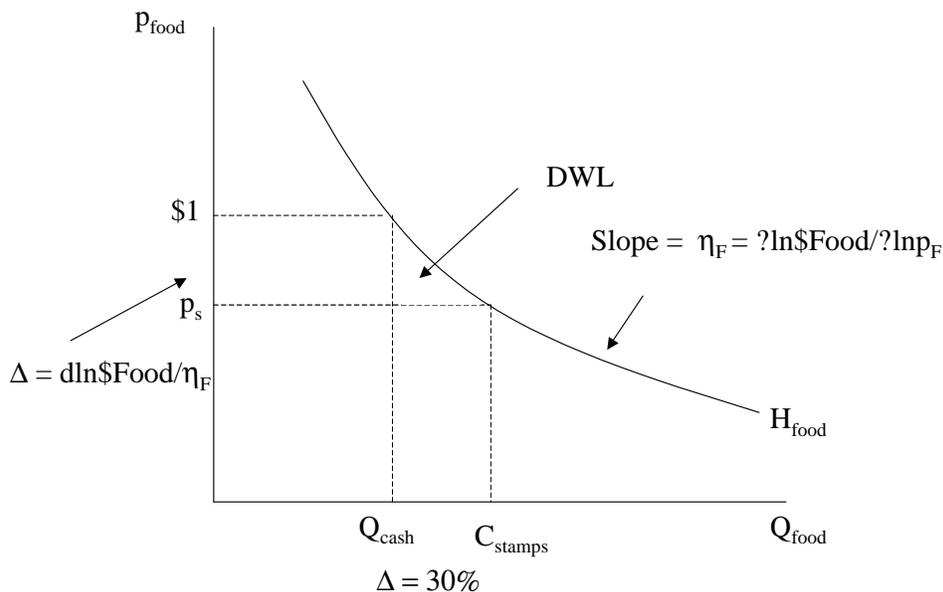
- Note the counter-factual assumption here: the stamp households would have spent the same on food as the cash households except for the stamp restrictions.

3.3 Estimating the welfare loss

- Question: Is this amount $F_s - F_c$ equal to the welfare loss for these households?
- No. They still value the food by some positive amount, even if they would have preferred to spend the money on other goods.
- To calculate this, it's easy to see what's going on visually.
- Question: What's the marginal utility of \$1 food for 'non-distorted' consumers relative to all other goods? Answer: Must be \$1.00 at the margin.
- We know how much extra food the 'distorted households are consuming.' Roughly 1/3rd more than they want to.
- So, if we know the slope of the compensated demand function, we could figure out how much utility they are losing *in cash equivalent terms*. That is, how much more cash they'd need to be as well off as the households receiving cash transfers instead of stamps.
- Question: What's an upper limit on this amount? Answer: $F_s - F_c$. If we gave them all of the extra money they spent on food (and let them keep the food), they would be as well off. But in fact they'd be better off unless they placed no value on food at the margin.

- See figure below.

7#2



- Question: What is the name of the shaded area? Dead weight loss.
- We know the following:
 - Shadow value of the marginal food items for undistorted households in terms of other goods foregone. Question: What is it? Answer: This is \$1.00.
 - We know the difference in quantity of food consumed P_c . About 20-30% more.
 - We'd like to know the welfare loss for 'distorted' households in cash equivalent terms.
- To get this, we need the Compensated Demand Elasticity for food, $\eta_f = \frac{\partial \ln \$Food}{\partial \ln P_f}$.
- **Why compensated, not uncompensated? Because we are trying to figure out how to make the consumer as well off.**

- So, plug in from some existing studies. It's in the range of -0.16 to -0.28. That is a 10 percent increase in food prices reduces demand by 2 to 3 percent (it's inelastic).

- Notice from the definition of the elasticity :

$$\eta_f = \frac{\partial \ln \$Food}{\partial \ln P_f} \Rightarrow \partial \ln P_f = \frac{\partial \ln \$Food}{\eta_f}$$

- So we can solve for the change in shadow value of marginal food items using the above, which is the change in height of the demand curve at the two quantities using a linear approximation to the elasticity:

$$\partial \ln P_f \simeq \frac{0.3}{-0.2} = -1.5$$

- Now, we can approximate the area of the DWL triangle (in dollar terms) as:

$$DWL = \frac{1}{2} \cdot \partial \ln \$Food \cdot \left(\frac{\partial \ln \$Food}{\eta_f} \right) \simeq -\frac{1}{2} \left(\frac{0.3^2}{0.2} \right) = -0.225$$

- Hence, on average \$0.23 of value is lost on each dollar of food stamp consumption over the non-distorted amount.

Another way of saying this is that extra food stamps beyond the desired amount of food are worth about \$0.77 apiece on average.

- SEE WHITMORE TABLE 4. She finds marginal dollar values of stamps of \$0.72 to \$0.95 per distorted consumer.

- Since distorted households spend about 30 percent more on food than cash households,' the estimated average percentage loss in value per distorted food stamp household is

$$\left(\frac{ExtraFood}{Q_{\$stamps}} \right) \cdot 0.23 = 0.30 \cdot 0.23 \simeq 0.07,$$

so \$0.07 on every dollar is wasted for distorted households.

- What is the total welfare loss? There were \$19,000 million in food stamps distributed in 2002. 20 percent of recipients are distorted, and their welfare

loss is \$0.07 on the dollar. Hence, $DWL \approx 19,000 \cdot 0.20 \cdot 0.07 = \266 million.

- Bottom line: the welfare loss is about one-quarter of a billion dollars, which is not enormous. [Whitmore estimates a value closer to half a billion dollars. Her approximation is preferable, but we don't want to go through all of the methodology to get there].
- A more positive way to put this result. The USDA could 'cash out' food stamps and simultaneously reduce benefits by $\sim \$250$ million without making recipients worse off. Or, it could 'cash out' food stamps and keep benefit levels the same and make recipients $\sim \$250$ million better off ($\sim \$15$ per recipient).
- If cashing out also reduced administrative costs, this would also be an important savings.
- If it shut down the underground market (eliminating transfers to criminals), this would increase targetting efficiency – more of the benefits would go to recipients rather than crooked grocers.

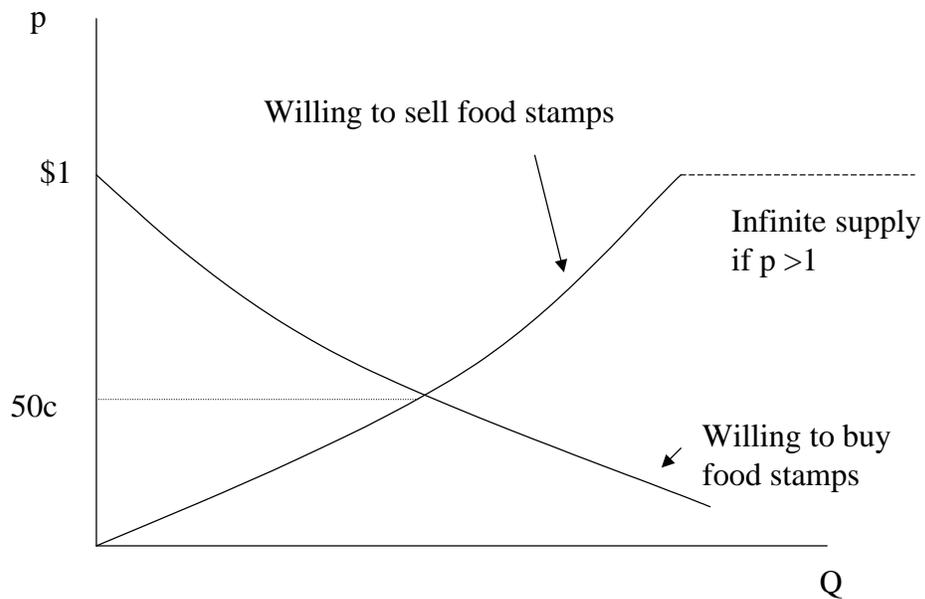
3.4 The underground market for food stamps

What these calculations neglect is the underground market.

- Since some consumers value food at less than \$1.00 at the margin while others do not, there are potential *gains from trade*. I can buy you food with my stamps, you can buy me something else.
- Moreover, grocers could in theory just give stamp holders cash (or sell them alcohol, cigarettes and other non-stamp goods) and redeem the stamps from the government at face value—though Electronic Benefits Transfer cards have made this harder.
- Will this market function efficiently such that stamps sell for \$1.00 each? No.

- First, this is fraud. Sellers could lose their stamp privileges, and buyers could be jailed.
 - So, buyers will demand a 'risk premium.'
 - Second, it takes time to find buyers. This means that the buyer does have to pay the seller the full face value because the seller will potentially have to search long and hard to find another buyer.
- Consequently, sellers will not get full face value. See figure 7#3. There will be a downward sloping demand curve of risk takers and an upward sloping supply curve of recipients who don't value food much at the margin. The intersection of these curves MUST be below \$1.00.

7#3



- But government will still pay \$1.00 per stamp to the grocer. So, part of the food stamp money is a transfer to crooks (though they would not necessarily have been crooks were it not for the program).
- What is the underground selling price?
- See Whitmore TABLE 6.

- “Survey Says:” Food stamps sell for about \$55 to \$65 dollar per \$100. Which is a large transfer to criminals.
- This survey *does not* tell us what share of stamps sold on black market since Whitmore could not ask this in survey (would not get reliable answers: “Hello, I’m calling from Princeton for a research project. Are you a crook?”).
- The USDA estimated in 1996-1998 that about 3.5% of every dollar of food stamps was trafficked.
- Note additional costs of trafficking:
 - Enforcement costs of reducing trafficking
 - People in jails (lead me not to temptation...)

3.5 Nutrition

These exercises above study how efficiently the food stamp program maximizes the utility of participants. In fact, this is not the program goal. The goal is to,

“...safeguard the health and well-being of the nation’s population by raising levels of nutrition among low-income households.”

So, perhaps the relevant question is whether cash transfers do a better or worse job than food stamps.

Question: What would you predict?

Question: What is the greater ‘nutrition’ problem facing most households – not enough calories, or too many?

- SEE TABLE 8C.
- Largest expenditure reductions are for:
 - Vegetables
 - Fruit

- Meat
- Legumes
- Juice and soda
- No impact on alcohol consumption.
- No data on cigarette consumption.
- SEE TABLE 9A.
- In terms of “Recommended Daily Allowances:”
 - Reduction in calories from 126% to 119% of RDA. *On average, people still overeating, but less so.*
 - On average, all RDA’s still met.
- Looking at the share meeting the RDA:
 - 10 percentage point reduction in share eating 100% of RDA of calories.
 - 3 percentage point reduction in share eating 200% of RDA of calories.
 - Only other categories that appear affected are Iron and Calcium.
- So, the evidence on nutrition is not crystal clear but it’s quite possible that reduction in calories outweighs reductions in other nutrients A lot of marginal money appears to be going into soda and juice.
- TABLE 10A.
- Some evidence that check recipients spend more on Utilities and Housing. That’s not necessarily bad.

3.6 Conclusions

1. Are recipients “distorted?” That is, do they indeed spend more on food than they otherwise would without food stamps?

- Yes they are. About 7 cents is wasted on the dollar.
2. What share of food stamps are sold onto the black market? And at what price?
 - About 3.5 percent of food stamps are trafficked illegally.
 - They sell at 50 to 60 percent at face value.
 3. Does cash versus in-kind have any effect on nutrition?
 - Does reduce calories.
 - Not clear it harms nutrition.
 4. How costly are cash versus in-kind programs to administer?
 - Cash versus EBT: EBT is about \$2.16 more expensive per person per month than sending checks.
 - Nationally, that's about \$200 million per year.
 - Retailers also spend about \$260 million per year to administer EBT.
 - Plus the cost of the underground market (harder to assess)

Other considerations:

- Political economy:
 - Food stamps have political support that welfare does not have because they are *not* viewed as a handout.
 - They have lobbying clout because they are administered by the Department of Agriculture and the Farm lobby seems to believe that food stamps are ultimately spent on farm products – and so Farmers view it as their subsidy too.
 - Possible that cashing out the program would help recipients in the short run, harm them in the long run.
- Other?