14.03 Fall 2004

Problem Set 6

Professor: David Autor

Due Friday, December 3, 2004, 5pm (with automatic extension to Monday, December 6, 2004 at 5 pm, if desired)

1 Moral hazard and insurance

Donald is a risk-averse person who has \$100 in monetary wealth and owns a house worth \$300. The probability that his house is destroyed by fire (equivalent to a loss of \$300) is $p_{ne} = 0.5$. If he exerts an effort level $e = 0.3$ to keep his house safe, the probability falls to $p_e = 0.2$. His utility function is:

$$U = w^{\frac{1}{2}} - e$$

where e is effort level exerted (zero in the case of no effort and 0.3 in the case of effort).

1. In the absence of insurance, does Donald exert effort to lower the probability of fire?

   Donald is considering buying fire insurance. The insurance agent explains that a home owner's insurance policy would require paying a premium α and would repay the value of the house in the event of fire, minus a deductible D. [A deductible is an amount of money that the company does not reimburse in the event of a loss.] The company explain that, without a deductible, insured clients do not take adequate precautions to reduce the possibility of fire. Unfortunately, these actions are not observed by the company and monitoring their insurees' behavior would be extremely costly. Hence, the insurance company cannot require effort as part of the policy.

2. Verify that the company is correct that, without a deductible, insurees would not take precautions against the possibility of fire. (Without a deductible, the company would reimburse the entire value of the house in the event of a fire)? Answer the question comparing expected utility under the two action choices ($e = 0$ or $e = 0.3$).

3. For a given α, what is the minimum amount of deductible D that would induce Donald to take precautions that lower the risk of fire?

4. Indicate how you would find the optimal level of α and D that maximize the insurance company profits and that make Donald indifferent between buying and not buying insurance (still assuming that the insurance company wants to induce high effort on the part of the insuree). [Do not try to solve it as you would need to solve complicated implicit functions].

5. Imagine that the the insurance company sold a policy with no deductible [$D = 0$]. Would Donald choose to buy an *actuarially fair* policy with no deductible if offered? Explain why or why not [calculations are required].

6. Consider that most insurance polices (auto, home, health) have mandatory deductibles; consumers are never fully insured. Is it correct to conclude that consumers would be better off with a law outlawing deductibles?

2 Rothschildia health plan

The government of Rothschildia (a small, densely populated island nation off of the coast of Kendall Square) is considering implementing a voluntary national health insurance plan. Everyone in Rothschildia is risk averse with VNM utility function $U(w) = w^{0.5}$. Each citizen has a wealth of 1 Stiglitz (the local curency) but should he or she become ill, s/he must spend her entire wealth of 1 Stiglitz on health care. (Since the cure is immediate and complete, the only disutility of illness is this 1 Stiglitz cost.) The only respect in which Rothschildians differ from one another is that each has a different *ex ante* probability of becoming ill, p_i. Illness probability is distributed uniformly among citizens on the interval $[0, 1]$, meaning that *on average* one-half of the population becomes sick in a year, but that each person has a different probability, p_i, ranging from zero to one where all values of are equally likely. If $p_i = 0$, person i is certain not to become ill, if $p_i = 1$, person i is certain to become ill, if $p_i = 0.5$, person i has a 50% probability of becoming ill, etc. Each citizen *knows* his own individual illness probability but the government only knows about the distribution of probabilities. Finally, assume that each citizen lives only 1 year but that a new generation is born every year (each also knowing his or her). The government (of course) persists from year to year.

Bear in mind the following about uniform probability distributions:

1. If a variable is distributed uniformly on the $[0,1]$ interval, the mean (i.e., expected) value of the variable greater than or equal to a given cutoff, κ, is $\frac{\kappa+1}{2}$. Similarly, the expected value *below* the cutoff, κ, is $\frac{\kappa}{2}$. Hence, the expected value of observations above $\frac{1}{2}$ is $\frac{3}{4}$, and the expected value of observations below $\frac{1}{2}$ is $\frac{1}{4}$.

2. If you want to calculate the expected value of a *function of a random variable*, you must integrate that function over the probability distribution of the random variable which is specified by its 'probability density function' (PDF).

The probability density function of a $U[0,1]$ (where U stands for uniform) variable could not be simpler. It is:

$$f(x) = 1$$

Meaning that all values between 0 and 1 are equally likely (and, moreover, the sum of probabilities of all possible values – the integral of $f(x)$ on $[0,1]$ is equal to 1).

For example, if $y = x^2$ and x is distributed $U[0,1]$, and you want to know the expectation of y, you would integrate the function over the PDF of x:

$$E(y) = E\left(x^2\right) = \int_0^1 x^2 \cdot 1dx = \frac{1}{3}$$

The problem

1. A policy maker for the Rothschildia Board of Health sits down to design the new national health plan. She reasons that since half of all Rothschildians get sick each year, the government should offer an actuarially fair full insurance policy that charges a premium of $\frac{1}{2}$ Stiglitz and pays a benefit of 1 Stiglitz to any enrollee who gets ill (the enrollee pays the premium regardless of whether or not she becomes ill). Given this premium, calculate who chooses to enroll in the plan:

   (a) What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

(b) What is the average health of those who enroll in the plan?

(c) Does the plan break even, make money, or lose money in year 1, and by how much per person on average?

2. In year 2, a different policy maker at the board of health notes that something went wrong in the first year: the plan made/lost money (depending on your answer above). He reasons, "Clearly we set the premium too high/low in the first year. What we'll do is set the new premium to reflect our average cost from last year. This should straighten things out."

   (a) What is the new premium?

   (b) What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

   (c) What is the average health of those who enroll in the plan?

   (d) Does the plan break even, make money, or lose money in year 2, and by how much (per person average)?

3. In year 3, a third policy maker observes that something is again amiss. The plan made/lost money again last year, although the intention was to break even. This policy maker suggests that the board fix the problem by setting the new premium at the average cost for year 2.

   (a) What is the new premium?

   (b) What is the expected illness probability of the most healthy and least healthy person to enroll in the plan?

   (c) What is the average health of those who enroll in the plan?

   (d) Does the plan break even, make money, or lose money in year 3, and by how much (per person average)?

4. In year 4, a fourth policy maker discovers that the plan has failed again. She says, "Alas, I see the error of our ways! Each time we change the premium, a different pool of citizens enrolls in the plan. I wonder if there is a premium we could set so that the pool of citizens who enrolls at that price costs us on average exactly that price. That way, we'd break even and provide insurance to all those who want to buy it." After a few strokes of the pen, she shouts, "Eureka! There it is."

What is that premium? Hint: you can either solve this problem analytically (i.e., on paper with a simple equation) or with a spreadsheet by repeating the steps you used for A, B, and C until you get a convergent solution.

5. In year 5, an economist from the Rothschildian Board of Social Welfare visits the Board of Health and says, "I see that you've worked out the national health plan premium so that it no longer makes/loses money every year. That's a step forward. However, I'm concerned that your break-even program is not actually maximizing average social welfare. What I'd like you to do is calculate average well-being (utility) under three different policy options: 1) No health plan; 2) Your current break-even plan; 3) A third mandatory plan for all citizens that also breaks even. Then report back to me."

    (a) Please perform these calculations for the three plans.

    (b) Which health plan do you recommend based upon your calculations?

6. Explain substantively why your preferred policy option yields higher average social utility than the other two health insurance plans.

3 The job market for Santa Claus

Aspiring Santa Claus candidates are selected at the North Pole every year. The Santa Claus hiring committee is faced with the following problem. There are two types of Santa Claus candidates: 40 percent of them ("Good Santas") can deliver $5,000$ gifts on Christmas eve. The other 60 percent ("Bad Santas") can deliver only $1,000$ gifts. The committee must establish pay *before* Santa Claus do their work – that is, before the committee can observe productivity. The productive Santa Claus candidates also differ from the unproductive ones in their cold weather endurance. Good Santas have a utility function over wage w and hours spent in the cold h:

$$U_G = w - h,$$

while Bad Santas have utility function:

$$U_B = w - 2h.$$

The time spent in the cold by the Santa Claus candidates does not affect their productivity but it does affect their well-being.

1. Draw a diagram in the (h, w) space and show the indifference curves for the two types of Santa Claus. In the same graph indicate the zero-profit conditions for the Santa Claus hiring committee, one for each type of potential Santa Claus. These zero-profit conditions are lines along which the difference between the wage and productivity is equal to zero, so $w = 1,000$ and $w = 5,000$.

2. If the committee could tell the candidate types apart, would any candidate spend time in the cold (other than when delivering presents to all of the good boys and girls, of course) to signal their level of productivity?

3. Assume that the committee feels strongly that it not overpay or underpay any specific worker (i.e., it wants each Santa's wage to equal his productivity). Given that the committee cannot tell the candidates apart, which Santa Claus candidates will spend time in the cold, and how much time will they spend? Show graphically the wage associated with the amount of time spent in the cold by the two types of candidates. Show the indifference curves for the two types of candidates associated with the contract they end up taking. Find the number of hours, h^*, spent in the cold that induce the committee to pay the high productivity wage.

4. As a result of asymmetric information, are the Good Santa Claus candidates worse off or better off than they would be with full information? How about the Bad Santas? Explain.

5. The melting polar ice cap has caused the hiring committee to do some rethinking. A member has the following proposal: "I've done some calculations and concluded that having Santas stand out in the cold is a dead weight loss. It would be more socially efficient if we paid all Santas the same wage, w^*, equal to their expected productivity." Show that the committee member's proposal is indeed more socially efficient than the old signalling method, in the sense that it raises average utility of the Santas.

6. The following year, a member of the Committee for Lobbying for All of the Unhappy Santas (CLAUS) appears before the hiring committee. She say that 40 percent of the CLAUS constituents are unhappy with the new (flat) wage. To protest, these 40 percent stand outside in the cold for h^* hours, where h^* is your answer to question 3. The committee has no choice but to pay the protestors the

high productivity wage (since they are revealed to be Good Santas). What does it pay the *non*-protestors?

7. The council member who proposed the flat wage in question 5 is summoned to explain why his proposal failed to yield a an equilibrium that could survive a joint deviation of all Good Santas (as in part 6), despite the fact that it was more socially efficient. Please provide a rigorous explanation.