

General Equilibrium in a Pure Exchange Economy

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1 Motivation

To now, we've talked about one market at a time: labor, sugar, food, etc.

But this tool is a convenient fiction. Not always a badly misleading fiction. But still a fiction.

Markets are always interrelated:

- Reduce sugar tariffs → reduce sugar prices
- → Drop in employment of sugar cane farmers
- → Cane workers apply for other farm jobs, depress wages for farm workers generally
- → Arable land is freed for other uses
- → Gives rise to new crop production
- → Prices of other farm products fall
- → Real consumer incomes rise
- → Rising consumer income increases demand for sweets, a luxury good
- → The dessert market grows and the café sector booms
- → Starbucks buys up all urban real estate in four major cities
- ...there is literally no end to this chain of events.

At some level, there is no such thing as the market for a single good.

All changes in quantities or prices feed back into the demand and/or supply for other goods through:

- Income effects
- Substitutability/complementarity of goods whose prices rise/fall

- Changes in the abundance/scarcity of other resources

To understand this story, we need a model that can accommodate the interactions of all markets simultaneously and determine the properties of the grand equilibrium. What we need to develop is a *general equilibrium* model, in contrast to the *partial equilibrium* models we've used thus far this term.

2 The Edgeworth Box

We need to reduce the dimensionality of the 'all markets' problem to something analytically tractable. But we need to retain the essence of the problem.

The Edgeworth Box (after Jevons Edgeworth) allows us to do this.

It turns out that we only need 2 interacting markets to see the entire problem:

- 2 goods
- 2 people
- Pure exchange.
- (No production in this model; we won't have time to develop this extension this semester, but it's worth understanding. If you want to understand this, ask for my lecture notes on general equilibrium with production.)
- Perfect expression of the economic concept of opportunity costs.

Simple as this model is, it demonstrates two of the most fundamental results in economics: the 1st and 2nd welfare theorems.

2.1 Edgeworth box, pure exchange

Two goods: call them food F and shelter S .

Two people: call them A and B .

The initial endowment:

$$E_A = (E_A^F, E_A^S)$$

$$E_B = (E_B^F, E_B^S)$$

Their consumption:

$$X_A = (X_A^F, X_A^S)$$

$$X_B = (X_B^F, X_B^S)$$

Without trade:

$$\begin{aligned}X_A &= E_A \\X_B &= E_B\end{aligned}$$

With trade, many things can happen, but the following must be true:

$$\begin{aligned}X_A^F + X_B^F &= E_A^F + E_B^F \\X_A^S + X_B^S &= E_A^S + E_B^S\end{aligned}$$

Note the elements of this figure:

- All resources in the economy are represented.
- The preferences of both parties are represented.
- The notion of opportunity costs is clearly visible.

Starting from point E , the initial endowment, where will both parties end up if they are allowed to trade?

It is not fully clear because either or both could be made better off without making either worse off. But it's clear that they need to be *somewhere* in the lens shaped region between U_A^0 and U_B^0 .

How do we know this?

Because all of these points **Pareto dominate** E : One or both parties could be made better off without making the other worse off.

There are potential gains from trade.

A would prefer more food and less shelter, B would prefer less food and more shelter.

So hypothetically

$$\begin{aligned}A \text{ gives up } &E_A^S - X_A^S \\A \text{ gains } &X_A^F - E_A^F \\B \text{ gives up } &X_B^F - E_B^F \\B \text{ gains } &E_B^S - X_B^S\end{aligned}$$

Are all points in the lens shaped region Pareto efficient?

No. All of the points in the lens region are Pareto superior, but only a subset are Pareto efficient.

Q: What needs to be true at a Pareto efficient allocation?

A: The indifference curves of A, B are tangent.

Why? Otherwise, we could draw another lens.

So trading should continue until a Pareto efficient allocation is reached.

Pareto efficient allocation:

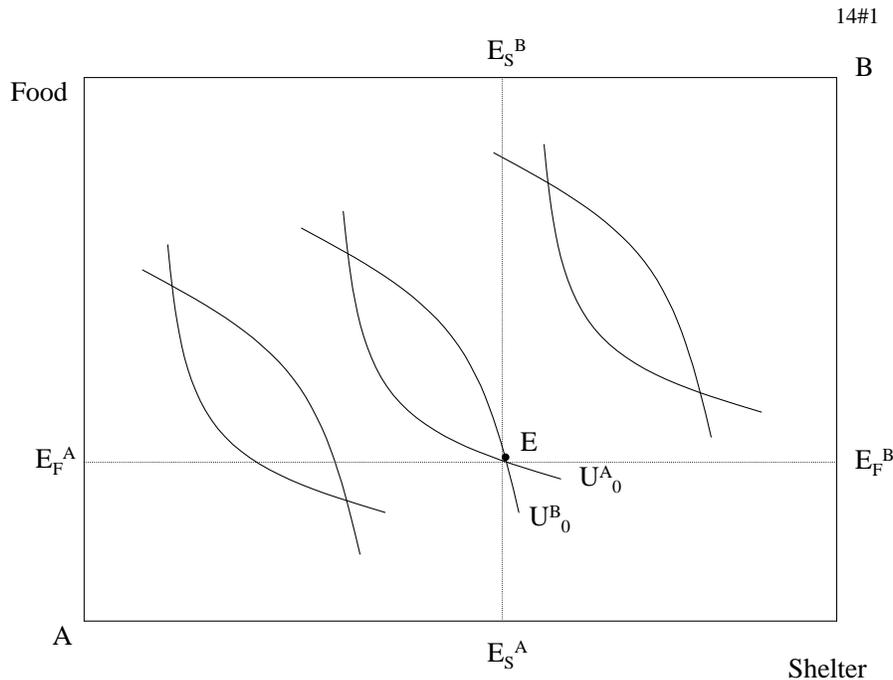
1. No way to make *all* people better off.
2. Cannot make 1 person better off without making at least 1 other person worse off.
3. All gains from trade exhausted.

At a Pareto efficient allocation, the indifference curves of A, B will be tangent.

[Except in the case of a corner solution. Imagine if A didn't like shelter and B didn't like food. There is only one Pareto efficient allocation in this case, and it is at a corner.]

The set of points that satisfy this criterion is called the Contract Curve (CC). All Pareto efficient allocations lie along this curve.

We know that after trade has occurred, parties' set of choices will lie somewhere on CC that passes through the lens defined by the points interior to U_A^0 and U_B^0 .



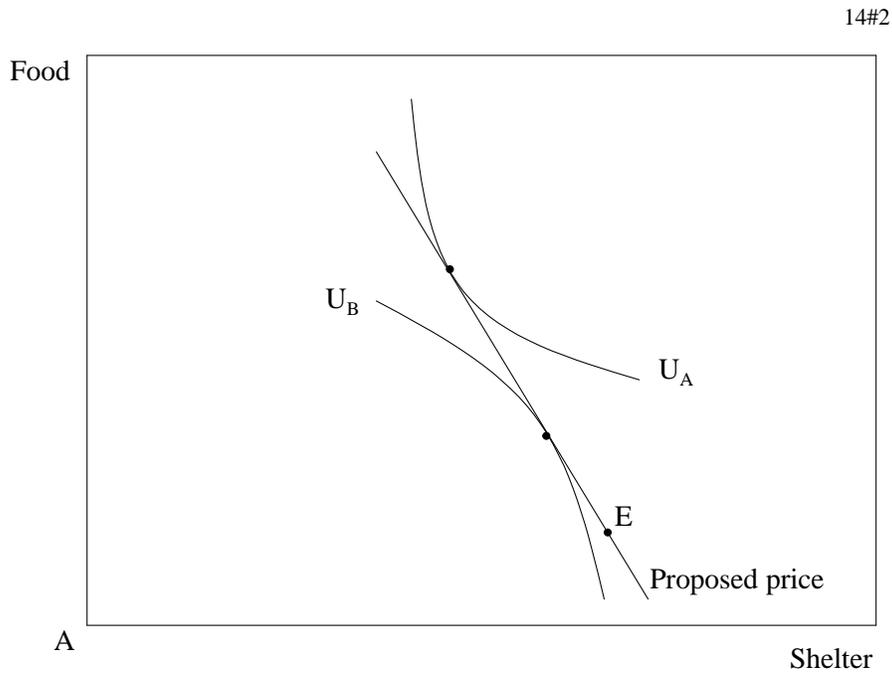
[In some examples, the Edgeworth box will not have a contract curve. That's because, for problems that yield a corner solution, there will likely be no points of tangency between the indifference curves of the two trading parties. But there may still be a set of Pareto efficient points (on the edges) that dominate the initial allocation. For example, if A values good X but not good Y and vice versa for B , there will be no tangency points and the only Pareto efficient allocation will involve giving the entire endowment of X to A and the entire endowment of Y to B .]

3 How do we get from E to a point on the contract curve?

Famous analogy: Auctioneer (Leon Walras \rightarrow Walrasian auctioneer).

1. In the initial endowment, *the market clears (that is, all goods consumed) but the allocation is not Pareto efficient.*
2. So, an auctioneer could announce some prices and then both parties could trade what they have for what they preferred at these prices.
3. Problem: *Choices would then be Pareto efficient but would not necessarily clear the market.*
4. It's possible there would be extra F and not enough S or vice versa.
5. So, must re-auction at new prices...

See Figure 2.



• At proposed prices:

- A wants to reduce consumption of shelter increase consumption of good
- B wants to increase consumption of shelter decrease consumption of food
- But, A wants to increase consumption of food more than B wants to decrease

- A wants to decrease consumption of shelter more than B wants to increase
- So:

$$X_A^F + X_B^F > E_A^F + E_B^F \Rightarrow \text{Excess demand}$$

$$X_A^S + X_B^S < E_A^S + E_B^S \Rightarrow \text{Excess supply}$$

How do we know that it is inefficient for some of the shelter to go unused?

What should auctioneer do? Raise P_F/P_S .

When the auctioneer gets the price ratio correct, the *market clears*. No excess demand or supply for any good.

This is a market equilibrium, competitive equilibrium, Walrasian equilibrium, etc:

- Each consumer choosing his most preferred bundle given prices and his initial endowment.
- All choices are compatible so that demand equals supply.
- Pareto efficient consumption ('Allocative Efficiency'):

$$\left(\frac{\partial U/\partial F}{\partial U/\partial S}\right)_A = \left(\frac{\partial U/\partial F}{\partial U/\partial S}\right)_B$$

Q: How do we know Allocative Efficiency will be satisfied?

- Because both A, B face the same prices P_F/P_S .
- Each person's optimal choice will therefore be the highest indifference curve that is tangent to her budget set given by the line with the slope P_F/P_S that intersects E .
- Because these choice sets (for A, B) are separated by the price ratio, we know they will be tangent to one another but will not intersect. (The set of prices forms a 'separating hyperplane' that divides the indifference maps of the two consumers).

This equilibrium price ratio *will exist* provided that:

- Each consumer has convex preferences (diminishing marginal rate of substitution) as we assumed during consumer theory.
- Or, each consumer is small relative to the aggregate size of the market so that aggregate demand is continuous even if individual preferences are not. (This is obviously not relevant in the two person case represented by the Edgeworth box.)

4 Aside: How do we know that both (all) markets clear simultaneously?

How do we know that both (all) markets clear simultaneously?

Consider two goods X, Y and two individuals A, B .

Label A 's demand and supply of each good as $D_x^A, D_y^A, S_x^A, S_y^A$ and similarly for consumer B .

Consumer A 's budget constraint can be written as:

$$\begin{aligned} P_x D_x^A + P_y D_y^A &= P_x S_x^A + P_y S_y^A, \\ P_x(D_x^A - S_x^A) + P_y(D_y^A - S_y^A) &= 0, \\ P_x ED_x^A + P_y ED_y^A &= 0, \end{aligned}$$

where ED_x^A is A 's 'excess demand' for good X . $ED_x^A = D_x^A - S_x^A$.

The excess demand is the amount of good A would like to consume relative to her current endowment (her 'supply').

Excess demands can be positive or negative (so more precisely, excess demand or excess supply).

The above equation states that given an initial supply (endowment) of goods and a set of prices, an individual's total excess demand for goods is zero. Simply put, a consumer cannot buy more than the value of the goods she holds, since the value of these goods is her budget constraint.

A similar budget holds for consumer B :

$$P_x ED_x^B + P_y ED_y^B = 0.$$

Putting these excess demand functions together,

$$\begin{aligned} P_x(ED_x^A + ED_x^B) + P_y(ED_y^A + ED_y^B) &= P_x ED_x + P_y ED_y = 0. \\ \text{and } P_x ED_x &= 0 \Rightarrow P_y ED_y = 0 \end{aligned}$$

Which is to say, that there cannot be either excess demand or excess supply for all goods simultaneously.

This observation – that total excess demand must equal zero – is called Walras' Law (after Leon Walras who first provided this proof).

So, if there are n goods, and there is no excess demand for $n - 1$ of these goods, then there is also *no excess demand* for the n^{th} good.

[We get the n^{th} solution for free because we have one more linear equations than unknowns. That's because we have one more goods than we have price ratios; good X , good Y , and price ratio p_x/p_y (as is obvious from the prior figure, it is only the *price ratio* not the absolute price level that matters). This fact implies that if we have n goods, the matrix of demands has rank $n - 1$. So, if we solve for market clearing prices of $n - 1$ goods, we have also obtained the market clearing price of the n^{th} good.]

In our two-good exchange economy above, this proves that if the market for food clears with no excess demand or excess supply, then the market for shelter clears simultaneously.

5 How are equilibrium prices set? First Welfare Theorem

You do not need the auctioneer. Leon Walras proved that the market can reach this equilibrium without assistance from a central planner (auctioneer). “The self-organizing economy.” Process of Tatonnement – translation ‘groping.’ This is a fundamental result. [I will not prove this. See your book.]

In partial equilibrium analysis, we have taken prices as exogenous. At the individual level, this is true. For all practical purposes, my preferences do not influence the price of sushi.

But in aggregate, prices depend on preferences. The market trade-off among goods – that is, the price ratio – depends on the aggregation of the psychic trade-offs among all potential consumers.

What Walras showed, and what is clear from the Edgeworth box, is that a competitive market will exhaust all of the gains from trade

If the following conditions are satisfied...

1. No externalities
2. Perfect competition
3. No transaction costs
4. Full information

then, the First Welfare Theorem guarantees that the market equilibrium will be Pareto efficient.

First Welfare Theorem: A free market, in equilibrium, is Pareto efficient.

5.1 Another way to see this:

We can think of the General Equilibrium problem as a utility maximization subject to three constraints:

1. No actor is worse off in the market equilibrium than in the initial allocation. How do we know this is satisfied? A person could always refuse to trade and consume her original endowment instead.
2. In equilibrium, no party can be made better off without making another party worse off (otherwise there are unexhausted gains from trade).
3. No more goods can be demanded/consumed than the economy is endowed with. That is, sum of the consumption of both parties cannot exceed the total endowment.

3a [No goods are left unconsumed – that is, there is no excess supply. This is not truly a constraint – it’s simply a property of any equilibrium, which follows from non-satiation.]

What the First Welfare Theorem says is that the Free Market Equilibrium is the solution to this problem. Simply by allowing unfettered trade among atomistic market actors, the market solution (i.e., the price vector and resulting equilibrium choices) will satisfy the three constraints above.

That’s a fairly amazing result. It implies that the decentralized market continually ‘solves’ a complex, multi-person, multi-good maximization problem that would probably be extremely difficult for any one individual (or large government agency) to solve by itself (due to the information requirements).

Of course, markets are not always (or necessarily ever) ‘in equilibrium.’ So, the market solution may not be perfect. But one should ask: would a ‘central planner’ generally do better? The answer is probably not – and it would be costly even to attempt it (again, due to the information requirements).

6 Second Welfare Theorem

Q: Does the 1st Welfare Theorem guarantee that the market allocation will be ‘fair’ or equitable?

No. Giving everything to A in the initial endowment would be Pareto efficient, as would giving everything to B .

The 1st Welfare Theorem simply says that the market will enlarge the pie as much as possible; it has nothing to say about who gets which slices.

So, is there a trade-off between enlarging and dividing the pie?

Stated rigorously, given a Pareto efficient allocation of resources, *will prices exist* such that this allocation is a market equilibrium?

The 2nd Welfare Theorem proves that the answer is yes.

Second Welfare Theorem: Providing that preferences are convex, any Pareto efficient allocation can be a market equilibrium.

The reasons are self-evident in the Edgeworth diagram (though this is a far from a proof).

- Along the contract curve, every point represents the tangency point of two indifference curves
- This tangency point corresponds to a price ratio that separates the two tangent indifference curves
- This price ratio clearly must exist if the indifference curves are tangent and each is convex (so they don’t recross at some later point)
- This price ratio is therefore the market price vector that will support that particular Pareto efficient allocation.

Hence, it is immediate from the Edgeworth box that all Pareto efficient distributions – that is, all points on the CC – are feasible as market equilibria.

As long as the assumptions above are met, a competitive equilibrium will exist merely because each person is self-interestedly maximizing her own well-being.

The Second Welfare Theorem says that *any* Pareto efficient allocation can be maintained as a competitive equilibrium.

This means that the problems of equity/distribution and efficiency can be separated.

Hence, another statement of 2nd welfare theorem is: *There is no intrinsic trade-off between equity and efficiency.*

[Notice that the converse is also generally true: non-Pareto efficient allocations cannot be attained in equilibrium.]

When we discussed consumer versus producer surplus in the Sugar case, I asserted that it was justified to maximize the sum of the two rather than worrying about their division. The 2nd welfare theorem is what justifies that assertion.

7 Conclusions

The fundamental welfare theorems provide some very basic policy guidance:

- The function of the price mechanism is to ensure that all resources are consumed in a Pareto efficient fashion – all gains from trade are exhausted.
- This occurs automatically as prices adjust to clear the market.
- Distorting the price system to achieve equity is intrinsically a bad idea (as we discussed in the partial equilibrium taxation example prior to the first mid-term). That’s because distorting the price system truly does create a trade-off between efficiency and equity – which is exactly what the Welfare theorems say we do *not* need to do.
- This does not mean we should ignore equity, however. We can achieve whatever ‘equitable’ allocations of resources is desirable through lump-sum distributions.

Is this dictum – don’t distort prices – always correct? No. Because the strong assumptions underlying the Welfare Theorems are not always – or perhaps ever – satisfied.

But it does build a prima facie case that free market outcomes may be efficient – or at least hard to improve upon.

But improving on them requires a careful analysis of why they are *not* desirable; and preferably a proposed solution that harnesses the efficiency properties of markets rather than attempting to override them.

When there is a case to be made for manipulating market outcomes (and there often is), this case probably should depend upon:

- A reasoned diagnosis as to why the market allocation is not optimal.
- A policy prescription that builds on an analysis of how a specific intervention will remedy this fault.
- A careful accounting of the likely distortions (deadweight losses) that will result from tampering with the price system.

8 Aside: How do we know that the welfare theorems are non-obvious?

This insight – that the free market system generates a Pareto efficient equilibrium – is *non-obvious*. Why would anyone assume that prices are other than arbitrary social creations?

And in fact in most of human history, prices and market operations have been viewed with a great deal of suspicion.

But economic theory suggests that market equilibria:

- Have a fundamental logic
- This logic is an *emergent property* of the rational, atomistic actions of market participants.

The key result: Blind pursuit of self-interest by autonomous actors in a market setting yields *collectively welfare maximizing behavior*. Under certain (strong) assumptions, this equilibrium *cannot be improved upon* without making at least one person worse off (Pareto efficiency).

Adam Smith published *The Wealth of Nations* in 1776. It's clear that Smith intuitively understood the First Welfare Theorem in *The Wealth of Nations*. For example, he wrote:

“It is not from the benevolence of the butcher, the brewer, or the baker, that we expect our dinner, but from their regard to their own interest. We address ourselves, not to their humanity but to their self-love, and never talk to them of our necessities but of their advantages.”

And:

“Every individual necessarily labors to render the annual revenue of the society as great as he can. He generally indeed neither intends to promote the public interest, nor knows how much he is promoting it. ... He intends only his own gain, and he is in this, as in many other cases, led by an invisible hand to promote an end which was no part of his

intention. ... By pursuing his own interest he frequently promotes that of the society more effectually than when he really intends to promote it. I have never known much good done by those who affected to trade for the public good.”

Clearly, Smith has convinced himself of the first welfare theorem (not obvious that he thought about the 2nd).

But it was 150 years until either welfare theorem was proved.

- Pareto and Barone proposed the 1st and 2nd welfare theorems formally in the 1930s.
- These theorems were proved graphically in 1934 by Abba Lerner.
- They were proved mathematically by Oskar Lange in 1942 and Maurice Allais in 1943 (for which Allais won the Nobel Prize in Economics in 1988).

Yet, for most of history, market behavior has been viewed with great suspicion.

For example:

In 1639 in Boston, the respected merchant Robert Keayne is charged with a heinous crime: *He has made over sixpence profit on the shilling, an outrageous gain.*

The Boston court debates whether to excommunicate him for his sin.

In view of his spotless past, the court fines him 200 pounds instead (a huge sum!).

Keayne is so distraught over his sin that he prostrates himself before the church elders and “with tears acknowledges his covetous and corrupt heart.”

The minister of Boston cannot resist the opportunity to make an example of Keayne.

In his Sunday sermon, he uses the example of Keayne’s avarice to denounce “some false principles of trade:”

1. That a man might sell as dear as he can, and buy as cheap as he can.
2. If a man loses by casualty of sea, etc., in some of his commodities, he may raise the price of the rest.
3. That he may sell as he bought, though he paid too dear.

[From Helibroner (1953), *The Worldly Philosophers* (New York: Touchstone).]

The hypothesis that free markets self-organize to produce socially desirable outcomes is a fundamental insight of economics. 225 years after Smith wrote *The Wealth of Nations*, this idea is still barely understood outside of the Economics profession (though it has gradually had a profound effect on the organization of modern economies).